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Integer Programming for Optimal Control of Geostationary Station Keeping of Low-Thrust Satellites

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Abstract: A control scheme is elaborated to perform the station keeping of a geostationary satellite equipped with electric propulsion. The use of electric thrusters imposes to take into account some additional mutually exclusive constraints on the control function that can be reformulated as logical constraints. The resulting fuel optimal station keeping problem is thus not solved with classical methods, either direct or indirect, but is transformed into a linear integer programming problem. The linearised relative velocity of the satellite is computed and some linear constraints on this velocity are added to the original station keeping problem in order to perform the station keeping over one week after another. Simulation results validate the efficiency of the optimal control thrusts obtained by the solution of the overall linear integer programming problem.

Keywords: Satellite control, GEO satellite station keeping, electric propulsion, integer programming.

1. INTRODUCTION

Satellites operating in Geostationary Earth Orbit (GEO) naturally drift outside their station keeping (SK) window (a rectangular box of a given geographical longitude and latitude range) due to orbital disturbing forces. Performing an accurate SK strategy is therefore necessary by using electric and/or chemical thrusters in order to counteract the effect of orbital perturbations.

Chemical propulsion systems have been and are still widely used. For these propulsion systems with high thrust capabilities, SK control laws are usually designed assuming an impulsive idealisation of the thrust, as described for example in Soop (1994). Electric propulsion for station keeping has been used since the late sixties (see, e.g. the work of Barrett (1967) and Hunziker (1970) ) and some theoretical developments have been presented in the eighties by Anzel (1988) and Eckstein (1980). Nowadays, the SK of GEO satellites can be performed reliably by electric as well as by the chemical propulsion. The bigger specific impulse of electric thrusters leads to a consequent savings in fuel consumption, what enables a reduction of the satellite mass and increased payload capacity and/or improved satellite lifetime.

Considering these technological and operational features, optimal control strategies for electrical SK, taking various constraints into account (minimum elapsed time between two consecutive firings, on-off profile of the thrusters, thrust allocation) have to be carefully designed. Some references as for example Sidi (1997), Campan et al. (1995) or Soop (1994) have established some rule of thumb based on the physical analysis of the effect of the external forces in order to perform the SK. In general, the problem of station keeping can be expressed as an optimal control problem even if the above mentioned constraints, inherent to the use of electric thrusters, prevent us to solve it with classical methods.

Optimal SK control problems may be solved according to several approaches. When simplifying assumptions are used, analytical control laws may be obtained, as in Sukhanov and Prado (2012). Otherwise, it is in general necessary to resort to numerical methods, such as direct collocation based methods as described in Hull (1997) and Betts (1998). For this kind of approaches integrating the satellite dynamic, the state and the control variables are discretised to produce a non linear programming problem and get an optimal open loop control. To deal with on-off models of the thrusters, rectangular profiles may be generated from a continuous one with the Pulse Width Modulation technique (see Vazquez et al. (2015) and the references therein). Losa et al. (2005) has formulated a method based on differential inclusion and Losa et al.
model (cf. Campan and Brousse (1994)) and \( \mathbf{f}_G \in \mathbb{R}^{6 \times 3} \) is the Gauss contribution part.

In order to deal with the station keeping problem, the relative state of the satellite with respect to the station keeping state:

\[
\mathbf{x}_{sk} = [a_{sk} \ 0 \ 0 \ 0 \ \ell_{MO_{sk}}]^T,
\]

is defined, where \( a_{sk} \) is the synchronous semi-major axis and \( \ell_{MO_{sk}} \) is the station mean longitude.

The relative dynamics are developed by linearisation of Equation (2) about the station keeping point (3). By denoting \( \mathbf{x} = \mathbf{x}_{eoe} - \mathbf{x}_{sk} \) the relative state model reads:

\[
\dot{\mathbf{x}} = A(t)\mathbf{x} + D(t) + B(t)\mathbf{u},
\]

where the matrices:

\[
A(t) = \frac{\partial}{\partial \mathbf{x}_{eoe}} \left( \bar{\mathbf{f}}_L(\mathbf{x}_{eoe}(t), t) \right) \Big|_{\mathbf{x}_{eoe} = \mathbf{x}_{sk}} \in \mathbb{R}^{6 \times 6},
\]

\[
B(t) = \mathbf{f}_G(\mathbf{x}_{sk}, t) \in \mathbb{R}^{6 \times 3},
\]

\[
D(t) = \mathbf{f}_L(\mathbf{x}_{sk}, t) \in \mathbb{R}^6,
\]

are obtained from the linearisation of functions \( \bar{\mathbf{f}}_L \) and \( \mathbf{u} \mapsto \mathbf{f}_G \mathbf{u} \).

As opposed to what is done in the literature, the linearisation is not performed with respect to an equilibrium point, but around a fictitious geostationary point. In other words, the equinoctial orbital elements \( \mathbf{x}_{eoe} \) undergo all disturbing external forces whereas the geostationary equinoctial elements \( \mathbf{x}_{sk} \) evolve following a keplerian motion. These two different dynamics for the state vector and the station keeping state vector explains the term \( \mathbf{D}(t) \) in the relative state model (4).

Recalling that 4 thrusters are available to realize the control, the transcription of the station keeping problem expressed in terms of the control vector \( \mathbf{u} \) would require to solve an allocation problem to find a right combination of thrusts. An alternative consists in considering directly the 4 thrusters provided by the 4 engines in the satellite dynamic. The control \( \mathbf{u}(t) \) can be written as a linear combination of the 4 thrusts such that \( \mathbf{u} = \Gamma \mathbf{F} \), where \( \mathbf{F} \in \mathbb{R}^4 \), \( \Gamma \in \mathbb{R}^{1 \times 4} \). In order to model the on-off nature of the control profile, the thrust vector \( \mathbf{F} = F_{\text{max}} [F_1 \ F_2 \ F_3 \ F_4]^T \) is composed of four normalized variables scaled by \( F_{\text{max}} \), the maximum thrust delivered by each propeller. The thrust direction matrix \( \Gamma = [\Gamma_1 \ \Gamma_2 \ \Gamma_3 \ \Gamma_4]^T \in \mathbb{R}^{3 \times 4} \) is defined such that each thrust direction \( \Gamma_j \in \mathbb{R}^3 \) are given by:

\[
\Gamma_j = \frac{1}{m} \left[ -\sin \theta_j \cos \alpha_j - \sin \theta_j \sin \alpha_j - \cos \theta_j \right]
\]

where angles \( \theta_j \) and \( \alpha_j \) are defined exactly as in Anzel (1988). Finally, the relative state model for the addressed SK problem is given by:

\[
\dot{\mathbf{x}} = A(t)\mathbf{x} + \mathbf{D}(t) + B(t)\Gamma \mathbf{F},
\]

The geographical coordinates of the satellite:

\[
\mathbf{y}_{eoe} = T(\mathbf{x}_{eoe}(t)) \mathbf{x}_{eoe},
\]

are the variables of interest since the station keeping problem consists in constraining them in the vicinity of
and the latitude of the satellite in a box defined by its size

Problem

2.2 Station Keeping as a Constrained Optimal Control

keeping amounts to minimizing the following performance

solution is looked for to extend the operational life time of

In this context, optimality means that a minimum fuel-

obtained by solving an associated optimal control problem.

via the 4 thrusters. An open loop control law can thus be

on a fixed time horizon by acting on the orbital parameters

the satellite stays in the geographical box. This box is

centered on the station keeping geographical position. By

denoting \( y = y_{\text{eoe}} - y_{sk} = T(x_{sk}, t)x = C(t)x \),

by linearising Equation (10).

The relative geographical velocity of the satellite is com-

puted by linearising the derivative of the state

evolution equation: \( \dot{y} = y_{\text{eoe}} - y_{sk} = T(x_{sk}, t)x = C(t)x \),

\( y = y_{\text{eoe}} - y_{sk} = T(x_{sk}, t)x = C(t)x \),

(11)

2.2 Station Keeping as a Constrained Optimal Control

Problem

The objective of the system is to maintain the longitu-

d and the latitude of the satellite in a box defined by its size \( \delta \)

on a fixed time horizon by acting on the orbital parameters

via the 4 thrusters. An open loop control law can thus be

obtained by solving an associated optimal control problem.

In this context, optimality means that a minimum fuel-

solution is looked for to extend the operational life time of

the satellite. Therefore, performing minimum-fuel station-

keeping amounts to minimizing the following performance

index under the dynamic constraint (9):

\[
\dot{J} = \int_0^T \sum_{\text{thruster } i=1}^4 \left( |u_R(t)| + |u_{T_i}(t)| + |u_{N_i}(t)| \right) dt
\]

\[
\Rightarrow J = \int_0^T \sum_{\text{thruster } i=1}^4 \left( |\Gamma_i| \right) dt.
\]

2.2.1 Station Keeping Requirements (Orbital Constraints)

Station keeping constraints are imposed to ensure that the

satellite stays in the geographical box. This box is defined in the plane (latitude,longitude) of width \( 2\delta \times 2\delta \)

centered on the station keeping geographical position. By

denoting \( y = y_{\text{eoe}} - y_{sk} \), the station-keeping constraints

over a finite time interval can be written as:

\[
|[0 \ 1 \ 0]y(t)| \leq \delta \quad \text{and} \quad |[0 \ 0 \ 1]y(t)| \leq \delta \quad \forall t \in [0, T].
\]

(16)

The initial condition is given by \( x(0) = x_0 \) whereas the

final state \( x(T) \) is free. The constraints on the geo-

graphical position may not suffice to perform the station keeping

over several time intervals. For instance, the final position

at \( t = T \) may be close to the boundary of the SK window

with the satellite velocity pointing outward. In such a case,

it may not be possible for a limited-propulsion satellite to

remain in the SK window for the ensuing time interval.

For this reason, a constraint on the final velocity of the

geographical variables is added:

\[
|[0 \ 1 \ 0]\dot{y}(T)| \leq \nu \quad \text{and} \quad |[0 \ 0 \ 1]\dot{y}(T)| \leq \nu,
\]

(17)

where the parameter \( \nu \) is chosen small.

2.2.2 Operational Constraints on Actuation

Beside the station keeping geographical constraints and

the usual bounds on the maximum thrust, technological

operational constraints on the actuation have to be taken

into account:

(a) actuators can only provide on-off thrusts;
(b) thrusters cannot be active simultaneously;
(c) each thrust must last at least \( T_1 \) (s);
(d) two successive thrusts of a given thruster must be

separated of an interval of latency equal to \( T_s = mT_1 \)

(s with \( m \in \mathbb{N}^+ \);
(e) two thrusters of different thrusters must be sepa-

rated by an interval of latency equal to \( T_d = T_1 \) (s).

Finally, the SK optimal control problem reads:

\[
\min_{\mathcal{F}(t)} J = \sum_{i=1}^4 \int_0^T F_i(t) dt,
\]

such that:

\[
\dot{x} = A(t)x + D(t)B(t)\Gamma F(t),
\]

\[
|x(0)| = x_0,
\]

\[
|\dot{y}_2(T)| \leq \nu, \quad |\dot{y}_3(T)| \leq \nu,
\]

initial condition,

final conditions

\[
|\dot{y}_2(t)| \leq \delta \quad \text{and} \quad |\dot{y}_3(t)| \leq \delta \quad \forall t \in [0, T]
\]

SK. const.

conditions (a) – (e)

control const.

(18)

3. INTEGER FORMULATION OF THE SK

PROBLEM

As proposed earlier, the Optimal Control Problem (OCP)

(18) will be addressed in the sequel by means of a direct

methodology. Note that solving (18) with indirect methods

by applications of the classical optimality conditions (Pon-

tryagin principle) is difficult due to the complexity of the

operational constraints involved and the presence of state

constraints. In the proposed methodology, the particular

disjunctive structure of the operational constraints is in-

vestigated in order to transform them into linear integer

constraints. The whole station keeping problem is then

recast as a linear integer programming problem and solved

with dedicated solvers.

3.1 Control Parametrization and Operational Constraints

We first parameterize the control profiles and then dis-

cretize the constraints in order to obtain an integer linear

programming problem.

As mentioned before in condition (a), the electrical engines

only produce on/off thrusts so that \( \mathcal{F}_i \) at time \( \tau_i \) can be

considered to be a 4-dimensional binary vector expressed by:

\[
\mathcal{F}(t) = [s_1 \ s_2 \ s_3 \ s_4]^T,
\]

(19)
where $s_l \in \{0, 1\}$, $l = 1, \ldots, 4$.

The time interval $[0, T]$ is divided in $N$ equal intervals $[\tau_i, \tau_{i+1}]$ with:

$$0 = \tau_0 < \tau_1 < \cdots < \tau_{N-1} < \tau_N = T. \quad (20)$$

The thrust vector $F(t)$ will be considered constant over each interval $[\tau_i, \tau_{i+1}]$, so that $F(t)$ is completely parameterized by the vectors $\{F_1, \ldots, F_N\}$:

$$F(t) = F_i, \quad \text{if } t \in [\tau_{i-1}, \tau_i], \quad i = 1, \ldots, N. \quad (21)$$

In order to cope with condition (e), the length of each interval is chosen such that $\tau_{i+1} - \tau_i = T_i$.

Beside this, two different thrusters cannot thrust simultaneously according to conditions (b). Thus, the control profile of the satellite can be considered at each time to be a binary word that has to be chosen among the five possibilities:

$$\forall i, \quad F_i = \begin{bmatrix} (s_1)_i, & (s_2)_i, & (s_3)_i, & (s_4)_i \end{bmatrix} \in \begin{bmatrix} 0 & 1 & 0 & 0 \ 0 & 0 & 1 & 0 \ 0 & 0 & 0 & 1 \ 1 & 0 & 1 & 0 \ \end{bmatrix}. \quad (22)$$

With such a parametrisation of the control, it is possible to write the operational constraints (b)-(e) as logical constraints. Let us consider a discretisation $\{\tau_i\}$ of the time interval $[0, T]$ in sub-intervals of length $T_i$. Defining $F_i = S(\tau_i)$, the constraints (c) is automatically satisfied. The constraint (d) can be transformed to:

$$\forall i, \quad [(F_i \neq 0) \& (F_{i+1} = 0)] \Rightarrow [(F_{i+2} \neq F_i)] \& (F_{i+3} \neq F_i) \& \ldots \& (F_{i+m} \neq F_i)]. \quad (23)$$

The constraint (e) can be expressed as:

$$\forall i, \quad F_i \neq 0 \Rightarrow F_{i+1} = \{0, F_i\}. \quad (24)$$

### 3.2 State Transition and States Constraints Transcription

Using the transition matrix of the differential equations of the satellite state vector evolution (9) as defined in Antsaklis (2003), the state vector trajectory, the geographical positions and velocities can be evaluated at each discretisation point: $\forall j = 0, \ldots, N$,

$$\begin{align*}
\dot{x}_j &= \Phi_j \dot{x}_0 + \sum_{i=1}^{j} \Phi_{ji} [B_{i}^F F_i + D_{i}^F (\tau_j)], \\
\dot{y}_j &= C_j \Phi_{j, \tau_0} \dot{x}_0 + \sum_{i=1}^{j} C_j \Phi_{ji, \tau_0} [B_{i}^F F_i + D_{i}^F], \\
\dot{y}_j &= E_j \Phi_{j, \tau_0} \dot{x}_0 + \sum_{i=1}^{j} E_j \Phi_{ji, \tau_0} [B_{i}^F F_i + D_{i}^F] + C_j [\bar{B}_i F_j + \bar{D}_i],
\end{align*} \quad (25)$$

with $x_j = x(\tau_j)$, $y_j = y(\tau_j)$, $\dot{y}_j = \dot{y}(\tau_j)$, $C_j = C(\tau_j)$, $D_j = D(\tau_j)$, $E_j = E(\tau_j)$, $\Phi_{ji, \tau_0} = \Phi(\tau_i, \tau_0)$, $\Phi_{ji} = \Phi(\tau_i, \tau_j)$, $\Phi_{ji, \tau_0} = \Phi(\tau_i, \tau_0)$, $\bar{B}_i = \int_{\tau_{i-1}}^{\tau_i} \Phi(\tau_i, s) B(s) ds$ $F_{i+1}$ $\Gamma$ $F_{i+1}$ $B_i$ and $D_{i}^F = \int_{\tau_{i-1}}^{\tau_i} \Phi(\tau_i, s) D(s) ds$.

This leads to the following discretisation of the SK constraints (16) and of the velocity constraints (17):

$$\begin{align*}
\sum_{i=1}^{N} b C_j \Phi_{ji} B_{i}^F F_i &\leq \delta - b C_j \Phi_{ji} x(0) - b C_j D_{i}^F, \\
- \sum_{i=1}^{N} b C_j \Phi_{ji} B_{i}^F F_i &\leq \delta + b C_j \Phi_{ji} x(0) + b C_j D_{i}^F, \\
\sum_{i=1}^{N} b E_N \Phi_{N_i} B_{i}^F F_i &\leq \nu - b E_N \Phi_{N_i} x_{0} - b E_N D_{i}^F - b C_N D_N, \\
- \sum_{i=1}^{N} b E_N \Phi_{N_i} B_{i}^F F_i &\leq \nu + b E_N \Phi_{N_i} x_{0} + b E_N D_{i}^F + b C_N D_N,
\end{align*} \quad (26)$$

with $\beta$ being either $[0 1 0]$ or $[0 0 1]$. The discretisation of the SK requirements are rewritten in matrix format as:

$$\begin{align*}
B_i F &\leq d - B_2 x(0) - B_c, \\
- B_i F &\leq d + B_2 x(0) + B_c, \\
G_i F &\leq \nu - G_2 x(0) - G_c, \\
- G_i F &\leq \nu + G_2 x(0) + G_c,
\end{align*} \quad (27)$$

where $F = [F_1, \ldots, F_N]^T$. With the transformation of the thrusters operational constraints of Section 2.2.2 and the previous discretisation of the dynamics equation, it is possible to use an integer programming formulation to solve the minimum fuel SK problem.

### 3.3 Actuators Constraints and Integer Programming

Considering that the parameter vectors $F_i$ are composed of binary variables $s_i$ (cf. (19)), the constraint (b) is expressed as:

$$\forall i, \quad (s_1)_i + (s_2)_i + (s_3)_i + (s_4)_i \leq 1. \quad (28)$$

Constraint (c) is satisfied by definition of the control parametrization. In order to express constraints (d) and (e), an auxiliary binary variable $\gamma_i \in \{0, 1\}^N$ is used and is defined as:

$$\forall i, \quad (s_1)_i + (s_2)_i + (s_3)_i + (s_4)_i = \gamma_i. \quad (29)$$

The variable $\gamma_i$ expresses the fact that one of the four thrusters is firing on interval $[\tau_i, \tau_{i+1}]$. If $\gamma_i = 0$, all thrusters are off. Thanks to $\gamma_i$ the constraint (d) reads:

$$\forall k = 1, \ldots, m, \forall i = 1, \ldots, N - k - 1, \forall l = 1, \ldots, 4, \quad (s_l)_{i+k} + (s_l)_{i} + \gamma_i - \gamma_{i+1} \leq 2, \quad (30)$$

and the constraint (e) is given by:

$$\forall i = 1, \ldots, N - 1, \forall l = 1, \ldots, 4, \quad (s_l)_{i+1} - (s_l)_{i} + \gamma_i \leq 1. \quad (31)$$

The fuel-consumption is expressed with the control profile $S$ by:

$$J_s = \sum_{i=1}^{N} [1 1 1 1] F_i, \quad (32)$$

and the final SK linear integer programming problem is defined by:
In this section, simulation results obtained with the proposed methodology are presented. Let us consider a satellite of mass 4850 kg equipped with 4 electric thrusters oriented in the directions North-East, North-West, South-East and South-West. This satellite has to be controlled in order to remain close to its geostationary position at a fixed longitude $\bar{\lambda}$ and a fixed latitude $\bar{\varphi} = 0$. The SK problem has been solved with Gurobi (see Gu et al. (2010)) and the Matlab parser Yalmip (see Löfberg (2004)) for a time horizon $T = 1$ week. The initial relative geographical position is $y(0) = [0.03^\circ -0.04^\circ]^T$ and the relative initial geographical velocity is $\dot{y}(0) = [0 0 0]^T$.

The benefits of the constraints on the geographical variables derivatives in the integer formulation (33) are checked by solving the SK problem on one hand with the final constraint on the geographical variables derivatives and on the second hand without the final velocity constraints. On Figure 1 and 2, the geographical positions for two cases defined above are drawn. It is clear that these differences in the design have a high impact on the final trajectory, as well as on the control profile (see Figure 3).

In order to perform the station keeping one week after another, the position in the SK window at the end of a week is used as the initial position for the following one. If the constraints on the geographical variables derivatives at that time are not included in the optimisation stage, the satellite trajectory may end near the boundary of the SK window with an outward pointing velocity, so that the optimisation problem can be infeasible the week after. For the above initial conditions, this phenomenon occurs at week 3. If the constraints on the derivatives of the geographical parameters at the final time are added, the station keeping problem can be solved week after week, but the price to pay is a fuel consumption increase by three times. With the integer formulation of the SK problem, final velocity constraints are much easily handled than with the Pontryagin Maximum Principle involved in Gazzino et al. (2016). Therefore, it is much simpler to optimise over several weeks with the proposed linear integer programming formulation.

It is possible to find in the literature some rules of thumb for geostationary SK strategies, in particular in Campan et al. (1995), Soop (1994) and Sidi (1997). In these references, the effect of the perturbing forces on the orbit is studied, and some general SK laws are derived. The North-South effect of the Sun and the Moon attractions are the most pregnant forces and these forces must be corrected each half orbit, once in the North direction and half an orbit later in the South direction. The East-West drift is meanwhile corrected by setting different thrust durations for each thrusters. This SK strategy was usefully used in an industrial context by Anzel (1995). The control profile of Figure 3 shows that the two South thrusters have a thrust at the beginning of each day, and the two North thrusters have a thrust half a day later. On the control profile, the thrusts have always different durations in order to compensate for the East-West drift. The physical rule of thumb can thus be recovered from a systematic optimisation process.
In this paper, the optimal control formulation of the minimum fuel station keeping problem has been transformed into a linear integer programming problem to cope with the on-off characteristics of the electrical thrusters and various operational constraints. The methodology has been illustrated with an example that highlights the interest of constraining the derivative of the geographical variable at the final moment of the optimisation horizon in order to prevent the satellite to fly out the station keeping window. The optimal SK strategy based on an optimisation method that could be derived in this paper matches the general framework derived by analysing the physical effect of the disturbing forces on the satellite orbit. The transition matrices used to build up the linear integer programming problem have been obtained under zero-holders assumption. However, in order to improve the accuracy of our results, further investigations have to be made to provide either closed-form expression or numerically certified approximations for the linearised dynamic.

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