An exact method for the Continuous Energy-Constrained Scheduling Problem with concave piecewise linear functions

Margaux Nattaf, Christian Artigues, Pierre Lopez

An exact method for the Continuous Energy-Constrained Scheduling Problem with concave piecewise linear functions

Margaux NATTAF[1,2], Christian ARTIGUES[1,2] and Pierre LOPEZ[1,2]

[1]CNRS; LAAS; 7 Avenue du colonel Roche, F-31077 Toulouse, France
[2] Univ de Toulouse, LAAS, F-31400 Toulouse, France
{nattaf,lopez,artigues}@laas.fr

Keywords: continuous scheduling, continuous resources, concave piecewise linear functions, energy constraints, energetic reasoning.

1 Introduction

This paper deals with a scheduling problem involving a set of tasks and a continuously-divisible renewable resource of limited capacity shared by the tasks. Each task must be processed between a release date and a deadline. During its time window, each task must receive a given total amount of resource units that we will refer to as a required energy amount. We consider the case where the resource amount (intensity) that a task requires during its processing is not fixed. More precisely, the resource requirement is a continuous function of time that must be determined. Once the task is started the resource usage must lie within an interval until the total required energy has been received by the task. Furthermore we consider that the total energy received by the task is not equal to the total amount of the resource used by it. Instead we have efficiency functions, which translates the required resource amounts into energy. Consequently, the duration of the task is not fixed neither but is determined by the resource requirement function as the task is finished once the necessary energy has been received.

As typical examples, we cite energy-consuming production scheduling problems. In (Artigues *et. al.* 2009), a foundry application is presented where a metal is melted in induction furnaces. Due to the complexity of the problem, efficiency functions were not considered in the paper. In a continuous time setting but still without considering the efficiency functions, constraint propagation algorithms based on the energetic reasoning concept were proposed in (Artigues and Lopez 2015). An extension of this work to linear efficiency functions were considered in (Nattaf *et. al.* 2015).

In this paper, we perform an analysis of the structural properties of the problem for more realistic concave piecewise linear functions (Hung *et. al.* 2005, Lewis 2009). We also adapt the energetic reasoning satisfiability test as well as the hybrid branch-and-bound algorithm developed for the linear case to the case of concave piecewise linear functions.

2 Problem definition

In the Continuous Energy-Constrained Scheduling Problem (CECSP), we have as input a set $A = \{1, \ldots, n\}$ of tasks and a continuous resource, which is available in a limited capacity B. Each task has to be performed between its release date r_i and its deadline $\tilde{d}_i$. Instead of being defined by its duration and resource requirement, in CECSP a task is defined by an energy requirement W_i, a minimal and maximal resource requirement b_i^{min} and b_i^{max}.

To solve the CECSP, we have to find for each task its start time st_i, its end time et_i and a function $b_i(t)$, for all $t \in \mathcal{T}$ (where $\mathcal{T} = [\min_{i \in A} r_i, \max_{i \in A} \tilde{d}_i]$), representing the amount of the resource allocated to this task. These variables have to verify the following equations:

$$r_i \leq st_i \leq et_i \leq \tilde{d}_i \qquad \forall i \in A \tag{1}$$

$$b_i^{min} \leq b_i(t) \leq b_i^{max} \qquad \forall i \in A; \forall t \in [st_i, et_i] \tag{2}$$

$$b_i(t) = 0 \qquad \forall i \in A; \forall t \in \mathcal{T} \setminus [st_i, et_i] \tag{3}$$

$$\int_{st_i}^{et_i} f_i(b_i(t))dt = W_i \qquad \forall i \in A \tag{4}$$

$$\sum_{i \in A} b_i(t) \leq B \qquad \forall t \in \mathcal{T} \tag{5}$$

where $f_i(b)$ is a continuous non-decreasing power processing rate function.

In this paper, we consider the case where functions f_i are non-decreasing, concave, piecewise linear and for which $f_i(0) = 0$. Therefore, let P_i be the number of pieces of function f_i and let $(\gamma_p^i)_{\{1,\ldots,P_i\}}$ be the breakpoints of f_i. Thus, $f_i(b) = a_{ip} * b + c_{ip}, \ \forall b \in [\gamma_p^i, \gamma_{p+1}^i]$. This problem is NP-complete (Nattaf *et. al.* 2015).

Now, we present a property of the CECSP, which will be helpful for solving it. Actually, we have proved that if a solution S exists, then another solution S' can be created from S with the property that each function $b_i(t)$ is piecewise constant. This is the statement of the following theorem:

Theorem 1. *Let $\mathcal{I}$ be a feasible instance of CECSP, with non-decreasing, concave piecewise linear functions f_i such that $f_i(0) = 0$, $\forall i \in A$. A solution such that, for all $i \in A$, $b_i(t)$ is piecewise constant, exists. Furthermore, $\forall i \in A$ the only breakpoints of $b_i(t)$ can be restricted to the start and end times of the tasks.*

The main idea of the proof is as follows. Let $(t_q)_{\{q=1..Q\}}$ be the increasing series of distinct start time and end time values ($Q \leq 2n$). Then, in each interval $[t_q, t_{q+1}]$, we set $b'_i(t)$ to the mean value of $b_i(t)$ over this interval, i.e. $b'_i(t) = \frac{\int_{t_q}^{t_{q+1}} b_i(t)dt}{t_{q+1}-t_q}$. The new solution consumes as much resource as the first one. For the energy received, we use Jensen inequality (Jensen 1906) to show each task receives an amount of energy greater or equal to the previous one. Finally, to obtain a feasible solution, we set, for each task receiving too much energy, its end time to the exact time where the task has received enough energy.

Now, we are able to present two satisfiability tests for the CECSP with concave and piecewise linear efficiency functions as well as a hybrid branch-and-bound to solve it.

3 Satisfiability tests for the CECSP

Energetic reasoning In this section, we present an extension of the energetic reasoning for the CECSP with non-decreasing, linear function f_i (Nattaf *et. al.* 2015) to the CECSP with non-decreasing, concave and piecewise linear f_i.

In order to present this satisfiability test, we define two quantities: the minimum resource consumption (resp. energy requirement) of a task i over an interval $[t_1, t_2]$, $\underline{b}(i, t_1, t_2)$ (resp. $\underline{w}(i, t_1, t_2)$). Then, the energetic reasoning consists in testing whether the available area within $[t_1, t_2]$ ($B \times (t_2 - t_1)$) is large enough to contain the minimum resource quantity needed by all the tasks in this interval.

To compute the minimum required resource consumption, we first have to compute $\underline{w}(i, t_1, t_2)$. Given an interval $[t_1, t_2]$, the minimum consumption always corresponds to a configuration where task i is scheduled at b_i^{max} outside interval $[t_1, t_2]$ or scheduled at b_i^{min} between $[t_1, t_2]$. Therefore, let I be the interval (note I can be the union of two intervals) over which i is scheduled at b_i^{max} outside interval $[t_1, t_2]$. Then, we have: $\underline{w}(i, t_1, t_2) = \max(f_i(b_i^{min})(t_2 - t_1)\,,\, W_i - |I| * f_i(b_i^{max}))$.

Then, in order to compute $\underline{b}(i, t_1, t_2)$, let $J = [t_1, t_2] \cap [r_i, \tilde{d}_i]$ be the interval over which task i must receive an energy $\underline{w}(i, t_1, t_2)$. First, since f_i is a non-decreasing function, processing a task i at b_i^{min} has the best efficiency ratio, i.e. $\max_{x \in [b_i^{min}, b_i^{max}]} f_i(x)/x = f_i(b_i^{min})/b_i^{min}$. We have two cases to consider:

- J is sufficiently large to schedule the task at b_i^{min}, i.e. $|J| \geq \frac{\underline{w}(i,t_1,t_2)}{f_i(b_i^{min})}$, and then $\underline{b}(i, t_1, t_2) = b_i^{min} \frac{\underline{w}(i,t_1,t_2)}{f_i(b_i^{min})}$
- J is not large enough to schedule the task at b_i^{min} and finding $\underline{b}(i, t_1, t_2)$ is equivalent to solving:

$$\begin{aligned} &\text{minimize } \int_J b_i(t)dt \\ &\text{subject to } \int_J f_i(b_i(t))dt \geq \underline{w}(i, t_1, t_2) \end{aligned}$$

In the latter case, using Jensen inequality (Jensen 1906), we know that it exists a solution where $b_i(t)$ is constant and this constant is equal to $f_i^{-1}(W_i/|J|)$. Since f_i is a non-decreasing concave piecewise linear function this value can easily be computed and, we have: $f_i^{-1}(W_i/|J|) = \max_{p \in P_i}(\frac{\underline{w}(i,t_1,t_2) - c_{ip}|J|}{a_{ip}|J|})$.

Time-table/Flow-based test In this section, we describe a linear program which helps to detect infeasibility. Indeed, if the program is infeasible, then there is no solution for the CECSP.

To describe this program, let $(t_q)_{q \in \mathcal{Q}}$ be the increasing series of distinct domain bounds of the start and end times values and let s_i^{max} (resp. e_i^{min}) be the latest start (resp. earliest end) time of i. Then, for each interval $[t_q, t_{q+1}]$ and for each task, we define two variables b_{iq} and w_{iq}, which stand for the quantity of resource (resp. energy) used (resp. received) by task i in this interval.

$$\sum_{i \in A} b_{iq} \leq B(t_{q+1} - t_q) \qquad \forall q \in \mathcal{Q} \tag{6}$$

$$b_{iq} \geq b_i^{min}(t_{q+1} - t_q) \qquad \forall i \in A\,;\forall q \in \mathcal{Q} | s_i^{max} \leq t_q \leq e_i^{min} \tag{7}$$

$$b_{iq} \leq b_i^{max}(t_{q+1} - t_q) \qquad \forall i \in A\,;\forall q \in \mathcal{Q} \tag{8}$$

$$b_{iq} = 0 \qquad \forall i \in A\,;\forall q \in \mathcal{Q} | t_q \notin [r_i, \tilde{d}_i] \tag{9}$$

$$w_{iq} \leq a_{ip} b_{iq} + c_{ip}(t_{q+1} - t_q) \qquad \forall i \in A\,;\forall q \in \mathcal{Q}\,;\forall p \in \{1..P_i\} \tag{10}$$

$$\sum_{q \in \mathcal{Q}} w_{iq} = W_i \qquad \forall i \in A \tag{11}$$

The linear program is based on the following observation: in all intervals $[t_q, t_{q+1}] \subseteq [s_i^{max}, e_i^{min}]$, task i has to be scheduled, at least, at b_i^{min}. These constraints are expressed by (7). Constraints (6) model the resource capacity constraints. Constraints (8) impose that the maximum resource requirement constraints are satisfied. Constraints (9) set the resource consumption of task i to be equal to 0 in $[t_q, t_{q+1}]$ if $[t_q, t_{q+1}] \not\subseteq [r_i, \tilde{d}_i]$. Constraints (10) combined with the objective function, ensure the resource conversion. Finally, constraints (11) state that the tasks received the required energy.

4 Hybrid branch-and-bound

In this section, we describe the hybrid branch-and-bound algorithm we used to solve the CECSP. This algorithm is an adaptation of the one in (Nattaf *et. al.* 2015) for linear functions.

First, a branch-and-bound algorithm is used to reduce the size of possible start and end intervals (until their size is less than a given $\epsilon > 0$) and, then, a start/end event-based MILP is used in order to find exact task start and end times and to determine the quantity of resource allocated to i between two consecutive events.

The branching procedure is inspired by the work of (Carlier and Latapie 1991). At the beginning, a task can start (resp. end) at any time $st_i \in [r_i, s_i^{max}]$ (resp $et_i \in [e_i^{min}, \tilde{d}_i]$). The idea is, at each node, to reduce the size of one of these intervals. For example, suppose that we choose to reduce the start time interval of i, then we create two nodes: one with constraint $st_i \in [r_i, (r_i + s_i^{max})/2[$ and one with constraint $st_i \in [(r_i + s_i^{max})/2, s_i^{max}]$. The variable on which we will branch is selected with the following heuristic: we choose the smallest interval among all $[r_i, s_i^{max}]$ and $[e_i^{min}, \tilde{d}_i]$.

At each node, we apply one or both of the satisfiability tests described above and, if the test does not fail, we perform the time-window adjustments proposed in (Nattaf *et. al.* 2015). We continue this procedure using a depth-first strategy until all intervals are smaller than ϵ. When it happens, the remaining solution space is searched via the event-based MILP.

Experiments have been conducted on the set of 129 instances of (Nattaf *et. al.* 2014) showing the interest of both satisfiability tests as well as the one of the hybrid branch-and-bound.

5 Conclusions

In this paper, we demonstrated that the resource usage changes can be restricted to start and end times of tasks. This allows us to present two polynomial satisfiability tests for the CECSP with concave piecewise linear power processing rate functions. We also presented a solution method including these satisfiability tests as well as a branching procedure and a Mixed Integer Linear Program.

In the continuity of this work, it will be interesting to establish the instance properties that make one test stronger than the other one. Studying more general functions is also a challenging research direction.

References

C. Artigues , P. Lopez , A. Haït , 2009, "Scheduling under energy constraints", *International Conference on Industrial Engineering and Systems Management (IESM 2009)*, Montréal, Canada.

C. Artigues and P. Lopez, 2015, "Energetic reasoning for energy-constrained scheduling with a continuous resource", *Journal of Scheduling*, Vol. 18(3), pp. 225–241.

J. Carlier and B. Latapie ,1991, "Une méthode arborescente pour résoudre les problèmes cumulatifs", *RAIRO. Recherche opérationnelle*, Vol. 25(3), pp. 311–340.

M.N. Hung, C. Le Van and P. Michel, 2005, "Non-convex aggregative technology and optimal economic growth", ftp://www-bsg.univ-paris1.fr/pub/mse/cahiers2005/B05095.pdf.

J.L.W.V. Jensen, 1906, "Sur les fonctions convexes et les inégalités entre les valeurs moyennes", *Acta Mathematica*, Vol. 30(1), pp. 175–193.

J. Lewis, 2009, "Algebra symposium: Optimizing fuel consumption", http://homepages.math.uic.edu/~jlewis/math165/asavgcost.pdf.

M. Nattaf, C. Artigues and P. Lopez, 2015, "Flow and energy based satisfiability tests for the Continuous Energy-Constrained Scheduling Problem with concave piecewise linear functions", *CP Doctoral Program 2015*, pp. 70–81, Cork, Ireland.

M. Nattaf, C. Artigues and P. Lopez, 2015. "A hybrid exact method for a scheduling problem with a continuous resource and energy constraints", *Constraints*, Vol. 20(3), pp. 304–324.