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To cite this version:
C Nainer, Michele Furci, Alexandre Seuret, Luca Zaccarian, Antonio Franchi. Hierarchical Control of the Over-Actuated ROSPO Platform via Static Input Allocation. Rapport LAAS n° 17074. 2017. <hal-01496283v2>

HAL Id: hal-01496283
https://hal.laas.fr/hal-01496283v2
Submitted on 23 May 2017

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Hierarchical Control of the Over-Actuated ROSPO Platform via Static Input Allocation *

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Abstract: This paper addresses the problem of control allocation applied to an over-actuated hovercraft-type vehicle. A hierarchical control architecture, consisting of a high level controller for trajectory tracking, and a control allocation algorithm, is developed and proved to be effective in tracking a desired trajectory while optimizing some cost related to actuator constraints. The control allocation algorithm exploits the redundancy of the system in order to keep the actuator states inside their saturation limits and tries to minimize the power consumption of the propellers. Unlike other papers on control allocation, actuator dynamics is taken into account. The control architecture is tested through simulations that well illustrate the capabilities of the proposed control design.

Keywords: Mobile Robots, Robot Control, Allocation, Over-actuated Robots, Hierarchical Control.

1. INTRODUCTION

Mini and micro Unmanned Aerial Vehicles (UAV) are being subject of many research topics in the last decade. The main motivations are the rich research problems they offer and the enormous amount of potential applications they have. Pertaining research topics include: control, motion planning, multi-agent systems, task allocation, coordination and many more. On the application side, we find: surveillance, industrial inspection, search and rescue, photography, 3D mapping, delivery and many more. A classical multirotor consists of a frame with 4 coplanar motor-propeller actuators that generate forces and torques. This is an under-actuated system, where the classical application consists of tracking a particular position trajectory, with the attitude dependent of the position trajectory. This can be a limitation for certain applications such as aerial manipulation, where the robot has to exert a specific force/torque in contact with the environment. Only in the last years researchers tried to overcome the limitation of planar multi-propeller UAV in terms of under-actuation by adding additional actuators. In (Odelga et al., 2016) the authors propose to use two extra servos to tilt the angle of the propellers and obtain a fully actuated quadrotor. In (Rajappa et al., 2015) six tilted propellers, with a fixed tilting angle can generate forces and torques in all directions, a property which is exploited in (Ryll et al., 2017) in order to perform interaction control with the environment. In (Ryll et al., 2016) an exa-rotor can tilt all the propellers in a coordinated fashion by means of an extra actuator, resulting in an overactuated system and finally in (Ryll et al., 2015) a quadrotor can rotate each tilt angle of the four propellers for an over-actuation. Additional recent examples of fully-actuated platform designs are provided in (Brescianini and D’Andrea, 2016) and (Park et al., 2016).

In all these works the inputs (propeller speed and tilt angle) are considered as ideal inputs, with no actuator dynamics and with unlimited bandwidth. These limitations can be however important when wanting to maximize the bandwidth of a certain closed-loop control action and should be taken into account in the control design. In particular, adding an extra actuator introduces new redundant input directions that can be exploited for a maximally effective use of the many actuators available. Allocation of redundant actuators has been a subject of intensive works in the past two decades, especially with reference to specific applications, such as the aerospace (Oppenheimer et al. (2006) and underwater Fossen and Johansen (2006) areas. Historical solutions deciding the optimal distribution of input authority among the available actuators typical rely on the online solution of an optimization problem, which can be computationally intensive, especially in light of the fact that different actuators are often in place to best deal with hard or soft limitations, which then result in nonlinear cost functions to be minimized (see, e.g., the scheme surveyed in Johansen and Fossen (2013) and Oppenheimer et al. (2010)).

In this paper we consider a simplified platform (the ROSPO platform) that can emulate the over-actuated multi-rotor configuration. The platform can also be used to emulate the cooperative manipulation of a load by means of n robots (Franchi et al., 2015; Petitti et al., 2016; Gioioso et al., 2014). It consists of a frame with a variable number of modules composed by a tilting turret and a motor-propeller actuator. As compared to a UAV, this platform is simplified because its dynamics is constrained to a 2D plane, having thus only 3 degrees
of freedom. Despite the simplified setting, this platform well represents the main challenges behind having a large number of actuators, thereby introducing redundancy and requiring some input allocation strategy. The goal is to test a control paradigm suitable merging a high level controller with a static input allocation technique for an over-actuated platform with actuators with a first order dynamics and limited input magnitude. In particular, we allocate the control action based on a scheme presented in (Passenbrunner et al., 2016), using suitable projection matrices capable of performing some kind of decoupling between the desired wrench to be applied to the physical system, and the cost function that should be optimized during operation. The scheme has the advantage of being computationally quite cheap, even though optimality is only guaranteed asymptotically. In this paper we show the effectiveness of the proposed control paradigm on the ROSPO platform. A control law on a similar platform but without the over-actuation was proposed by Aguiar et al. (2003). The over-actuation on 2D motions is instead a well studied problem for ships and vessels as in (Berge and Fossen, 1997) and as widely overviewed in Fossen and Johansen (2006); Fossen et al. (2009).

The paper is organized as follows: in Section 2 we present the platform and its dynamical model. In Section 3 we describe the control architecture for the system, composed by an allocation strategy and a high level controller for motion tracking. Finally in Section 4 we present some simulations for the platform with the proposed controller.

2. MODEL

Figure 1 shows the ROtor-graSPing Omnidirectional (ROSPO) platform schematic representation including the main symbol definitions. The system is composed by a rigid rectangular frame sustained omnidirectional passive spherical wheels and n actuating modules. Each module consists of a turret, which is orientable by means of a servo motor. Each turret carries a propeller driven by a BLDC motor. In the particular case shown in Fig 1 it is n = 3, however, the developed theory extends naturally to a generic number of turrets n ≥ 3.

The actuating modules can produce a thrust by spinning the propeller and can independently rotate the whole turret to orient the exerted force. The platform can move on the ground with very low friction thanks to “ball transfer unit” wheels. To describe the dynamical model of the platform we have to define two reference systems $\mathcal{W}$ and $\mathcal{B}$. $\mathcal{W}$ is the fixed ‘world’ inertial frame while $\mathcal{B}$ is the body frame whose origin is the Center of Mass (CoM) of the platform and whose axes are aligned with the platform itself.

The system dynamics can be defined by the following differential equations:

$$\begin{align}
\dot{\mathbf{p}} &= \mathbf{v} \\
n\dot{\mathbf{v}} &= \sum_{i=1}^{n} \mathbf{R}(\psi) \mathbf{f}^B_i \\
\dot{\psi} &= \omega \\
j \dot{\omega} &= \sum_{i=1}^{n} \mathbf{w}^T \mathbf{r}_i \mathbf{f}^B_i \\
\dot{\theta}_i &= u_{\theta,i} \\
\dot{w}_i &= u_{w,i} \\
\mathbf{f}^B_i &= k_w w_i^2 \frac{\cos(\theta_i)}{\sin(\theta_i)},
\end{align}$$

where $m$ and $j$ are constant scalars representing the total mass and moment of inertia of the system, $\mathbf{p} = [px\ py]^T \in \mathbb{R}^2$ is the position of the center of mass of the system in $\mathcal{W}$, $\mathbf{v} \in \mathbb{R}^2$ the velocity of the center of mass of the system in $\mathcal{W}$, $\psi \in \mathbb{R}$ is the yaw angle of the system defined as the angle between the x axes of $\mathcal{B}$ and $\mathcal{W}$, $\omega$ is the angular velocity of the platform, $\mathbf{R}(\psi)$ is the rotation matrix transforming the orientation from $\mathcal{B}$ to $\mathcal{W}$, $\mathbf{f}^B_i \in \mathbb{R}^2$ are the forces produced by each actuator module, expressed in $\mathcal{B}$, $\mathbf{r}_i \in \mathbb{R}^2$ are the positions of each actuator module w.r.t. the center of mass of the system, $\mathbf{w}^T \mathbf{r}_i$ is the spinning rate of the i-th actuator angle, $w_i$ is the spinning rate of the propeller of each actuator module, $w_{\theta,i}$ is the spinning acceleration, with $i = 1, \ldots, n$ and $k_w$ is a constant aerodynamic parameter. Equations (1a) can be referred to as the platform model, while equations (1b) as the actuators’ dynamics.

Note that we wrote the actuator dynamics (1b) by recognizing that an instantaneous variation of $\dot{\theta}_i$ or $\dot{w}_i$ is not possible. Therefore, it makes sense to consider $\dot{\theta}_i = u_{\theta,i}$ and $\dot{w}_i = u_{w,i}$ as the actual inputs. With this strategy we may well represent the rate limitations of the actuators as input constraints on $u_{\theta,i}$, $u_{w,i}$ and their maximum excursions as state constraints on $\dot{\theta}_i$ and $\dot{w}_i$. In particular we enforce:

$$\begin{align}
\dot{\theta}_i &\leq \bar{\theta}_i \\
\dot{w}_i &\leq \bar{\omega}_i \\
\dot{u}_{\theta,i} &\leq \bar{u}_{\theta,i} \\
\dot{u}_{w,i} &\leq \bar{u}_{w,i} \\
\dot{w}_{\theta,i} &\leq \bar{w}_{\theta,i} \\
\dot{w}_{w,i} &\leq \bar{w}_{w,i},
\end{align}$$

where all the underlined and overlined terms are constant scalar values. Constraints (2) imply that the spining rates of the propellers and the angle of the actuator turrets are both limited, as well as the inputs to control them (due to electro-mechanical and power limitations of the actuators). The angle of each turret is limited due to servo-motors limitations, while the maximum propeller speed is limited for power and dissipation reasons. Finally the minimum propeller speed is due to the fact that Electronic Speed Controllers (ESC) without hall sensors (namely most of the commercial ESC) do not perform well at low speed.

Equation (1) can be written in a more compact way as:

$$\begin{align}
\dot{\mathbf{p}} &= \mathbf{v} \\
n\dot{\mathbf{v}} &= \mathbf{R}(\psi) \mathbf{f} \\
\dot{\mathbf{w}} &= \mathbf{f} \\
\dot{\theta}_i &= u_{\theta,i} \\
\dot{w}_i &= u_{w,i} \\
\mathbf{f}^B &= k_w w_i^2 \frac{\cos(\theta_i)}{\sin(\theta_i)},
\end{align}$$

Fig. 1. Schematic representation of the ROSPO platform and main symbol definitions.
where \( f = \sum_{i=1}^{n} f_i^D \in \mathbb{R}^2 \) and \( r = \sum_{i=1}^{n} (\Pi r_i)^T f_i^B \in \mathbb{R} \) can be well understood as the total force and torque applied to the system by the actuators.

### 3. CONTROL ARCHITECTURE

In a typical task for this platform, we want to control a desired position and attitude trajectory. Hence the system has three regulated outputs (two positions and one angle), while the available inputs are \( 2n \). This means that for \( n \geq 2 \) the system is over-actuated, while for \( n = 1 \) the system is under-actuated. In this paper we consider the case \( n \geq 2 \). We can sum up the peculiarities of the system, which correspond to:

- Nonlinear plant;
- Over-actuated system;
- Actuators with nonlinear dynamics;
- Actuators with constrained state and input.

We want to develop a hierarchical control architecture consisting of a high level control for motion control (trajectory tracking) and a control allocation algorithm to handle the over-actuation in the actuators. In this architecture, the high level controller specifies a desired wrench to be coming from a high level controller.

#### 3.1 Static Allocation

We design the control allocation algorithm following Passenbrunner et al. (2016). We summarize briefly the fundamental of the corresponding allocation technique. Consider a virtual control \( \mathbf{u}_v \in \mathbb{R}^{n_v} \), a number of redundant actuators \( n_a > n_u \), and a commanded virtual control \( \mathbf{u}_{v,c} \in \mathbb{R}^{n_{v,c}} \) coming from a high level controller. The actuators should obey a first-order (possibly coupled) dynamics:

\[
\dot{x}_a = f(x_a) + g(x_a) u_v, \quad u_v = h(x_a)
\]

with \( f : \mathbb{R}^{n_a} \rightarrow \mathbb{R}^{n_a} \), \( g : \mathbb{R}^{n_a} \rightarrow \mathbb{R}^{n_a \times n_a} \), \( h : \mathbb{R}^{n_a} \rightarrow \mathbb{R}^{n_{v,c}} \).

In this platform we have \( n_a = 2n \) and \( n_{v,c} = 3 \).

The allocator generates \( u \) such that \( \lim_{t \rightarrow \infty} \| \mathbf{u}_v - \mathbf{u}_{v,c} \| = 0 \), while minimizing a cost function \( J : \mathbb{R}^{n_{v,c}} \rightarrow \mathbb{R} \).

In particular, the static allocation control is given by:

\[
u = g(x_a)^{-1}( -f(x_a) + u_v - \mathbf{u}_j ) \tag{5}\]

where:

\[
\mathbf{u}_v = \gamma_P \nabla h(x_a) \left( \nabla h(x_a)^T \nabla h(x_a) \right)^{-1} (\mathbf{u}_{v,c} - h(x_a))
\]

while \( \nabla (\cdot) \) indicating the gradient of a function and \( \nabla h(x_a) \) the following projection matrix:

\[
\nabla h(x_a) = I - \nabla h(x_a) \left( \nabla h(x_a)^T \nabla h(x_a) \right)^{-1} \nabla h(x_a)^T.
\]

The two terms \( \mathbf{u}_v \) and \( \mathbf{u}_j \) guarantee respectively the regulation condition and the optimality condition. The positive gain \( \gamma_P \in \mathbb{R}^+ \) is used to adjust the speed of the linear first order filter governing the dynamical relationship between \( \mathbf{u}_{v,c} \) and \( \mathbf{u}_c \). Instead, gain \( \gamma_R \in \mathbb{R}^+ \) is used to adjust the speed of the allocation term that minimizes cost \( J \).

For this specific application, according to (1), we can define:

\[
x_a = [w_1 \; \theta_1 \; \ldots \; w_n \; \theta_n]^T \tag{8}
\]

\[
u = [w_1 \; u_{\theta_1} \; \ldots \; w_n \; u_{\theta_n}]^T \tag{9}
\]

\[
f(x_a) = 0, \quad g(x_a) = I
\]

\[
h(x_a) = \begin{bmatrix}
\sum_{i=1}^{n} k_w w_i^2 \left( \frac{\cos(\theta_i) \sin(\theta_i)}{\sin(\theta_i)} \right) \\
\sum_{i=1}^{n} (\Pi r_i)^T k_w w_i^2 \left( \frac{\cos(\theta_i) \sin(\theta_i)}{\sin(\theta_i)} \right)
\end{bmatrix} \tag{11}
\]

A crucial aspect that influences the behavior and the optimality of the allocator relies on the choice of the cost function \( J \).

For our application the cost function was chosen as follows:

\[
J = \sum_{i=0}^{n} \left( \mu_1 \left( \frac{\tilde{w}_i}{\Delta_{w,i}} \right)^6 + \mu_2 \left( \frac{\tilde{\theta}_i}{\Delta_{\theta,i}} \right)^6 + \mu_w w_i^2 \right), \tag{12}
\]

with:

\[
\tilde{w}_i = w_i - w_{m,i}, \quad \tilde{\theta}_i = \theta_i - \theta_{m,i}
\]

\[
\Delta_{w,i} = \bar{w}_i - w_i, \quad \Delta_{\theta,i} = \bar{\theta}_i - \theta_i
\]

\[
\bar{w}_i = \frac{w_i + \bar{w}_i}{2}, \quad \bar{\theta}_i = \frac{\bar{\theta}_i + \theta_i}{2}
\]

and where \( \mu_1, \mu_2, \mu_w \) are positive scalar weights.

The first two terms in (12) impose a soft state constraint on the states of the actuators according to (2). The functions are indeed almost zero in the intervals \([\bar{w}_i, \bar{w}_i], [\bar{\theta}_i, \bar{\theta}_i]\) and very steep when approaching the state limits \([\bar{w}_i, \bar{w}_i], [\bar{\theta}_i, \bar{\theta}_i]\).

The third term in (12) penalizes high rotational speeds of the propellers, to reduce the energy consumption of the platform.

**Lemma 1.** If \( rank(\nabla h(x_a)) = 3 \), system (1b) augmented with allocator (5)-(11) is such that:
The goal is to design a high level controller that stabilizes the feedforward signal as a first order linear filter having a pole in $-\gamma p$. The equivalent system is represented in Figure 3 and can be written as:

$$
\begin{align*}
\dot{v} &= R(p)v_1 u_v, \\
\dot{w} &= S_2 u_v - \gamma p (v - v_e).
\end{align*}
$$

(15)

where $u_v, v \in \mathbb{R}^3$ are the states of the first order linear filter, $S_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $S_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

The goal is to design a high level controller that stabilizes (15) for a trajectory tracking controller. The desired trajectory can be expressed as $p(t), \dot{p}(t), \cdots, p^{(i)}(t)$, $v(t), \dot{v}(t), v_r(t)$, where $p(t) \in \mathbb{R}^2$ is a desired position trajectory in time, $v_r(t) \in \mathbb{R}$ a desired attitude trajectory and the subsequent functions are their time derivatives. We define the tracking error variables as:

$$
\begin{align*}
\tilde{p}(t) &= p(t) - p(t) \\
\tilde{v}(t) &= v(t) - v(t).
\end{align*}
$$

(16)

The goal of the high-level controller is to ensure $\lim_{t \to \infty} \tilde{p} = 0$ and $\lim_{t \to \infty} \tilde{v} = 0$, namely the trajectory converges to the desired path.

To this end, we propose the following selection:

$$
\begin{align*}
u_v &= B^{-1} (\tilde{u} - A \tilde{u}^*) - K \tilde{u}_v,
\end{align*}
$$

(17)

where $A = -\gamma p I_3$, $B = \gamma p I_3$ and $K \in \mathbb{R}^{3 \times 3}$ is a gain matrix such that $A - BK$ is Hurwitz. Selection (17) uses the feedforward signal

$$
\begin{align*}
\tilde{u}^* &= \begin{bmatrix} mR^T(\psi) (k_p \tilde{p} - k_D \tilde{p} + A \tilde{p} R) \\

\tilde{v} &= \begin{bmatrix} (k_p \tilde{v} - k_D \tilde{v} + A \tilde{v} R) \end{bmatrix}
\end{align*}
$$

(18)

with $\tilde{u}_v = u_v - u_v^*$ and where the four positive gains, $k_p \in \mathbb{R}^{+}$, $k_D \in \mathbb{R}^{+}$, $k_P \in \mathbb{R}^{+}$, $k_D \in \mathbb{R}^{+}$, assign a PD like behavior of the translational and rotational dynamics.

Controller (17), (18) ensures a desirable closed-loop behavior with the equivalent dynamics (15). This is established in the next lemma.

Lemma 2. Consider the feedback interconnection between plant (15) and controller (17), (18). For these dynamics, the point $(\tilde{p}, \tilde{v}, \tilde{\psi}, \tilde{u}_v) = 0$ is globally exponentially stable.

3.3 Overall Control Structure

The overall control input is defined as:

$$
[u_{v,1} \ u_{v,2} \ \cdots \ u_{v,n} \ u_{\theta,1} u_{\theta,2} u_{\theta,3}]^T = u_v - u_j
$$

(19)

with $u_v$ and $u_j$ defined in (6) and with a commanded virtual control $u_v, c$ defined as in (17).

With this overall scheme in place, the following main result can be proven by combining the results proven below in Lemmas 1 and 2.

Theorem 3. Consider some class $C^3$ functions $p_R(t), v_R(t)$ as desired position and attitude trajectories for the platform. Consider the overall closed loop system (1), (19), (6)-(11), (17), (18). Consider a cost function $J : \mathbb{R}^{n_a} \to \mathbb{R}$. The overall closed loop system is such that:

- The trajectories $p(t), v_R(t)$ of system (1) converge globally and exponentially to $p_R(t), v_R(t)$.
- The commanded virtual control $u_v, c(t)$ converges to $u_v, c$ that depends only by the reference trajectories and its derivatives.
- For any constant reference trajectory leading to a constant value of $u_v, c$, $x_a$ converges to a stationary point of $J(x_a)$, subject to $h(x_a) = u_v, c$.

In particular, if $J$ is strongly convex, then $x_a$ converges to the global minimum of (20).

4. SIMULATIONS

In this section we first describe the parameters of our system and then we present the main results of the simulations. All the parameters are taken from the real ROSPO platform and estimated by experiments. The experimental platform has $n = 3$ actuator modules, but is modular and other modules can be easily attached. For our system ((1a) and (1b)) we identified $m = 5.1$ kg, $j = 0.42 kgm^2$ and $k_w = 6.5 \cdot 10^{-4} Ns^2$ the inertia was estimated considering the mass of the platform structure and the mass of each single turret (0.6 kg). For each turret we estimated considering the mass of the platform structure and the mass of the turret itself (0.6 kg). The inertia was estimated considering the mass of the platform structure and the mass of each single turret (0.6 kg). The inertia was estimated considering the mass of the platform structure and the mass of each single turret (0.6 kg). The inertia was estimated considering the mass of the platform structure and the mass of each single turret (0.6 kg). The inertia was estimated considering the mass of the platform structure and the mass of each single turret (0.6 kg).
Finally, for the cost function (12) in the allocator block we had to consider the following physical constraints of the actuators: $\bar{w}_i = 95$ Hz, $w_i = 9$ Hz, $\bar{\theta}_i = 4\pi$ and $\theta_i = -4\pi$. The weight parameters in (12) have been selected as $\mu_1 = 300$, $\mu_2 = 750$ and $\mu_u = 1/2000$, in order to have comparable values of the penalty terms appearing in (12) and corresponding to the different soft saturations imposed by the allocation strategy. The cost function for a single actuator module can be seen in Figure 4, where one can appreciate its increasing behavior close to the soft actuator limits.

For the simulation we set the following allocation gains: $\gamma_P = 1$ and $\gamma_J = 4$. The reference trajectory for the high-level controller is an $\infty$-shaped track. The corresponding function is the following:

$$
\begin{align*}
\rho_R(t) &= \rho \cos(c_1 t) \\
\theta_R(t) &= \rho \sin(c_2 t) \\
\psi_R(t) &= c_3 t + \pi / 2
\end{align*}
$$

with $\rho = 4$, $c_1 = 0.2$, $c_2 = 0.4$ and $c_3 = -0.18$. The last equation of the reference $\psi_R$ for $\psi$ imposes a constant yaw rate along the $\infty$-shaped path.

The simulation step was set equal to 10 ms. We imposed an error at the initial position of the platform in order to show the transient behavior of our controller. Figure 5 shows in dashed line the desired path and in solid red line the actual path followed by the platform. The rectangles on the path show the position and orientation of the platform at some time instants. The three thin orange lines appearing on each rectangle show the orientation of the turrets. The length of those lines is proportional to the force generated by each module. Only the first 30 seconds out of the 150 of the simulation were plotted, to avoid making the plot too crowded and to better highlight the initial transient. These 30 seconds correspond to an almost complete “$\infty$” track.

5. CONCLUSIONS

We considered the control allocation problem in an over-actuated hovercraft vehicle. A hierarchical control archi-

| Preprint version, 5 20th IFAC World Congress, Toulouse, France (2017) |


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