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Observer-based event-triggered control for linear systems subject to cone-bounded nonlinearities

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Abstract: The paper presents an observer-based event-triggered control strategy for linear systems subject to input cone-bounded nonlinearities by using only available measurable variables. Sufficient conditions based on linear matrix inequalities are proposed to ensure the asymptotic stability of the closed loop and the avoidance of Zeno behavior in an emulation context. Based on these conditions, a convex optimization problem to compute the parameters of the event-trigger rule aiming at reducing the number of control updates is proposed. The approach is illustrated on a numerical example that considers the control of a linear system with a logarithmic input quantization constraint.

Keywords: Event-triggered control, cone-bounded nonlinearities, observer, stability, LMI.

1. INTRODUCTION

In the context of the implementation of modern control systems through digital communication networks, event-triggering strategies have been proposed mainly to deal with communication, energy consumption and computational constraints (see, for example, Postoyan and Girard (2015); Postoyan et al. (2015); Tabuada (2007); Heemels et al. (2012) and references therein.) In the event-triggered control framework, one can basically consider two approaches. The first one, which corresponds to the so-called emulation problem, considers that the controller is given a priori (see, for example, Heemels et al. (2012), Wang and Lemmon (2008), Postoyan et al. (2015), Seuret and Prieur (2011) Abdelrahim et al. (2016) and references therein). In this case, the focus is dedicated to the design of the event-triggering rule or function that leads to a stable closed-loop behavior while avoiding the occurrence of Zeno phenomenon. The second approach, which is referred as a co-design problem, performs the design of both the controller and the event-triggering rule, simultaneously, and is addressed in a few papers (Aström (2008); Seuret et al. (2013, 2016), Heemels et al. (2013); Abdelrahim et al. (2014); Antunes et al. (2012)).

A large amount of papers addressing asymptotic stability of event-trigger control schemes consider state-feedback laws (see for instance Tabuada (2007); Heemels et al. (2012); Lehmann and Lunze (2011b); Wu et al. (2014); Aranda-Escolastico et al. (2015); Wang and Lemmon (2008); Moreira et al. (2016)). In this case, it is in general easier to prove stability and that Zeno behavior does not happen. However, in most practical applications the full state measurement is not available or even possible. The fact of using available information (measured or local signals) in order to build the event-triggered control law,

from an emulation or a co-design point of view is a challenging problem: see, for example, Lehmann and Lunze (2011a), Almeida et al. (2012), Abdelrahim et al. (2016), Donkers and Heemels (2012), Tallapragada and Chopra (2012), Postoyan et al. (2015), Wang and Lemmon (2011), Heemels et al. (2012), Xia and Fei (2013) and references therein. On the other hand, we can also notice that most of the works in the literature consider linear plants. Some papers dealing with generic results for nonlinear systems are Tabuada (2007), Postoyan et al. (2015), Abdelrahim et al. (2016).

In the current paper we focus on the emulation design of event-triggered control for a particular class of nonlinear systems, which regards linear plants subject to input cone-bounded nonlinearities. This type of input nonlinearities can model, for instance, practical actuators nonlinearities or imperfections (e.g. saturation, deadzone, hysteresis, etc), as well as implementation control constraints such as quantization effects. Moreover, we are concerned by the use of only local information or measurable signals. To do so, a state observer-based approach is considered. In this context, we follow the same vein as in Tallapragada and Chopra (2012) to tackle the design of the event-triggering strategy for observer-based state feedback based on the decrease of a Lyapunov function. Recall that in the event-triggered control context, the plant evolves in continuous time, whereas the control signal is updated depending on discrete-time events. Then, the resulting closed-loop system can be cast as a hybrid or impulsive system. Instead of considering the classical hybrid framework to study mixed continuous and discrete dynamics as defined in Goebel et al. (2012), we use an alternative direction as proposed in Tarbouriech et al. (2016b), Tabuada (2007). In this case, sufficient conditions based on linear matrix inequalities (LMI) are proposed to ensure the global asymptotic stability of the origin of the nonlinear closed-loop system under the event-trigger control strategy. Moreover, following the idea presented in Mazo et al. (2010), Zeno behaviors are avoided thanks to the introduction of a minimum

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dwell-time, which is explicitly forced as a design parameter of the LMI conditions. These conditions are then cast in convex optimization problems to compute the parameters of the event-trigger rule aiming at reducing the number of control updates. The paper can then be considered as a comprehensive version of Tarbouriech et al. (2016b), since we extend the class of systems under consideration to some particular class of non-linear systems. The originality of the paper relies on the fact that the approach is based on the satisfaction of a continuous-time stability criterion as well as a discrete-time one, in which a minimum dwell-time is a direct tunable parameter.

The paper is organized as follows. In Section 2, the system under consideration is described, together with the problem we intend to solve. Conditions to solve the event-triggered control problem in an emulation context are proposed in Section 3 first in a generic format and second through linear matrix inequalities, in which the dwell-time appears as a tuning parameter. Section 4 provides a simple optimization criterion, in order to cope with the implicit objective of reducing the number of control updates by playing on the event-trigger rule parameters. In Section 5, an example illustrates the efficiency of the proposed approach. Finally, Section 6 presents some concluding remarks and directions for potential future works.

Notation. $\mathbb{Z}$, $\mathbb{R}^n$, $\mathbb{R}^{n \times n}$ denote respectively the sets of integers, n-dimensional vectors and $n \times m$ matrices. For any matrix A, A' denotes its transpose and $He\{A\} = A + A'$. For any square matrix A, $trace(A)$ denotes its trace. For two symmetric matrices of the same dimensions, A and B, $A > B$ means that $A - B$ is symmetric positive definite. I and 0 stand respectively for the identity and the null matrix of appropriate dimensions. For a partitioned matrix, the symbol $*$ stands for symmetric blocks. $\| . \|$ stands for the Euclidean norm.

2. PROBLEM STATEMENT

2.1 System data

Consider the following continuous-time plant:
$$\begin{cases} \dot{x}_p(t) = A_p x_p(t) + B_p u(t) + B_{pf} f(u(t)), \\ y_p(t) = C_p x_p(t) + D_{pf} f(u(t)), \end{cases} \quad (1)$$
where $x_p(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y_p(t) \in \mathbb{R}^p$ are the state, the input and the output of the plant, respectively. Matrices A_p, B_p, B_{pf}, C_p and D_{pf} are constant and of appropriate dimensions. Pairs (A_p, B_p) and (C_p, A_p) are supposed to be controllable and observable, respectively. Function $f(u) : \mathbb{R}^m \to \mathbb{R}^l$ is a nonlinearity affecting the input u. It can represent a neglected nonlinearity, uncertainty or imperfection affecting the input (Johansson and Robertsson (2002)). In other words, $f(u)$ allows to represent the error with respect to a linear input. For example, in the case of magnitude saturation of the input, $f(u)$ corresponds to the deadzone $sat(u) - u$ (see, for example Tarbouriech et al. (2011), Zaccarian and Teel (2011), Turner and Herrmann (2014)). Another interesting case, which can be described through $f(u)$, consists in capturing the fact that only a part of the input is saturated whereas the other part remains linear (see, for example, Tarbouriech et al. (2016a)). One can also cite the case of linear systems in the presence of some other isolated nonlinearities, such as quantization or backlash, which can also be represented by equation (1) (see, for example, Liberzon (2003), Tarbouriech et al. (2014), Jayawardhana et al. (2011) and references therein). This nonlinearity is therefore supposed to be continuous and to satisfy a cone-bounded property (see, for example, Khalil (1992), Castelan et al. (2008)). That means that this nonlinearity satisfies the following property
$$f(u)'S(f(u) + Ru) \le 0 \quad (2)$$
where $S \in \mathbb{R}^{l \times l}$ is any diagonal positive definite matrix. Matrix $R \in \mathbb{R}^{l \times m}$ is supposed to be a diagonal positive matrix that is fixed by the designer and depends on the nonlinearity characteristics. Moreover, it is assumed that Property (2) is globally satisfied, i.e. it is valid for all $u \in \mathbb{R}^m$.

In this paper, we consider an observer-based state feedback controller to stabilize the plant (1) and therefore to drive the output to zero. This controller is defined by:
$$\begin{cases} \dot{\hat{x}}(t) = A_p \hat{x}(t) + B_p u(t) + B_{pf} f(u(t)) - L(y_p(t) - \hat{y}(t)), \\ \hat{y}(t) = C_p \hat{x}(t) + D_{pf} f(u(t)), \\ u(t) = K \hat{x}(t), \end{cases}$$
$$(3)$$
where $\hat{x}(t) \in \mathbb{R}^n$ and $\hat{y}(t) \in \mathbb{R}^p$ are the state and the output of the observer, respectively. Furthermore, $L \in \mathbb{R}^{n \times p}$ and $K \in \mathbb{R}^{m \times n}$ are the observer and controller gains, respectively.

By considering the continuous-time system described by (3), the control design is carried out according to the separation principle. The observer gain L is then designed to make $A_p + LC_p$ Hurwitz. In this case, the state estimation error dynamics is given by
$$\dot{e}(t) = (A_p + LC_p)e(t), \quad (4)$$
where $e(t) = x_p(t) - \hat{x}(t) \in \mathbb{R}^n$, is globally asymptotically stable, i.e. $\lim_{t \to \infty} e(t) = 0$. Consequently, the estimation output error $e_y(t) = C_p e(t)$ also asymptotically converges to zero, i.e. $\lim_{t \to \infty} e_y(t) = 0$. On the other hand, the dynamics of the observer with the state feedback controller $u(t) = K\hat{x}(t)$ is given by:
$$\dot{\hat{x}}(t) = (A_p + B_p K)\hat{x}(t) + B_{pf} f(K\hat{x}(t)) - Le_y(t). \quad (5)$$

In this paper, we want to address the problem of an aperiodic event-triggered-based sampled-data implementation of such an observer-based controller. This paper can be viewed as an extension of the technique developed in Tarbouriech et al. (2016b) to the particular class of nonlinear systems represented by equation (1).

2.2 Sampled-data control implementation

We are interested in studying the way to implement the control input u. We consider that u is not continuously implemented but is updated at certain instants $\{t_k\}_{k \in \mathbb{N}}$, which form a sequence of strictly increasing positive scalars. The control action is supposed to be held constant between two successive sampling instants (t_k and t_{k+1}) through a zero order holder. Nevertheless, differently from classical digital control approaches, the sampling interval $t_{k+1} - t_k$ is not assumed to be constant but can be seen as an additional control action.

Thus, the closed-loop system can be represented by
$$\begin{cases} \dot{x}_p(t) = A_p x_p(t) + B_p u(t_k) + B_{pf} f(u(t_k)), \\ \dot{\hat{x}}(t) = A_p \hat{x}(t) + B_p u(t_k) + B_{pf} f(u(t_k)) - Le_y(t), \\ u(t_k) = K\hat{x}(t_k), \qquad \forall t \in [t_k, t_{k+1}). \end{cases} \quad (6)$$
Taking into account (4), performing a change of coordinates, the closed-loop dynamics (6) can be expressed in terms of $\hat{x}$ and e, as follows:

$$\begin{cases} \dot{\hat{x}}(t) = (A_p + B_p K)\hat{x}(t) + B_p \delta(t) \\ \qquad\quad + B_{pf} f(K\hat{x}(t) + \delta(t)) - Le_y(t), \\ \dot{e}(t) = (A_p + LC_p)e(t), \end{cases} \qquad (7)$$

where we use the same formulation as in Tabuada (2007) to define $\delta(t)$:

$$\delta(t) = u(t_k) - u(t) = K(\hat{x}(t_k) - \hat{x}(t)). \qquad (9)$$

This function $\delta(t)$ can be seen as a measure of the difference between the fictive continuous-time control value and its sampled and held version, which is currently implemented. Note that $\delta(t)$ depends only on the observer variables and is therefore available at the controller node.

2.3 Problem formulation

This work focuses on the event-triggered implementation of the controller represented by (3). Then, an event generator algorithm has to be included in the controller to decide whether or not the control input has to be updated, based on the available information. Adapting the event-triggered control strategy proposed in Tallapragada and Chopra (2012, 2013), the sampling instants are determined from the following logic:

$$t_{k+1} = \min\{t \geq t_k + T, \quad s.t. \quad g(\delta(t), y_a(t)) \geq 0\}. \qquad (10)$$

where y_a represents the vector of available information to the controller (which corresponds in our case to $y_a(t) = [\hat{x}(t)', e_y(t)']'$) and the function $g : \mathbb{R}^m \times \mathbb{R}^{n+p} \to \mathbb{R}$ has to be efficiently defined such that the asymptotic stability of the closed-loop system (7) under the event-triggered rule described in (10) is ensured. Moreover, note that the trigger criterion given by (10) ensures that the next sampling time will occur at least T time units ahead the last one. Thus, T can be seen as a minimal dwell-time, which allows to prevent Zeno solutions. For $t \geq t_k + T$ the control is not be updated until $g(\delta(t), y_a(t)) \geq 0$.

3. EVENT-TRIGGERED STRATEGY DESIGN

Our aim is to provide an event-triggered strategy to update the control signal applied to the plant based solely on available signals, that is, using only $u(t), f(u(t)), \hat{x}(t)$ and $y_p(t)$. According to (10), this corresponds to design T and g in order to ensure the asymptotic stability of the sampled-data system (6). To do so, we first provide a general formulation, inspired from Tallapragada and Chopra (2012), Tallapragada and Chopra (2013), which corresponds to extend Theorem 1 of Tarbouriech et al. (2016b). With this aim, consider the augmented state $x = [\hat{x}'\ e']' \in \mathbb{R}^{2n}$.

Theorem 1. Consider a positive scalar T, a function $g : \mathbb{R}^m \times \mathbb{R}^{n+p} \to \mathbb{R}$ and the triggering rule

$$t_{k+1} = \min\{t \geq t_k + T, \quad s.t. \quad g(\delta(t), y_a(t)) \geq 0\} \qquad (11)$$

with $y_a(t) = \begin{bmatrix} \hat{x}(t) \\ e_y(t) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & C_p \end{bmatrix} x(t)$.

Consider a positive definite function V, for which there exist two positive scalars ε_1 and ε_2 such that, for any $x \in \mathbb{R}^{2n}$, we have

$$\varepsilon_1 \| x \|^2 \leq V(x) \leq \varepsilon_2 \| x \|^2, \qquad (12)$$

and assume, in addition, that the function $V(x)$ satisfies, for all $t \in [t_k + T\ ,\ t_{k+1})$ and for all $k \in \mathbb{N}$

$$\dot{V}(x(t)) - g(\delta(t), y_a(t)) - 2f(u(t_k))'S(f(u(t_k)) + Ru(t_k)) < 0, \qquad (13)$$

and, for all $k \in \mathbb{N}$

$$\Delta V_T(x) - 2f(u(t_k))'S(f(u(t_k)) + Ru(t_k)) < 0, \qquad (14)$$

with $\Delta V_T(x) = V(x(t_k + T)) - V(x(t_k))$.

Then, the origin of system (7) with the triggering rule (11) is globally asymptotically stable and the inter-sampling intervals are lower bounded by T.

Proof. The proof is omitted due to space limitations but can be obtained from the authors. □

From Theorem 1, a way to design the event-triggered rule using only the available signals is stated below.

Theorem 2. Given controller and observer gains K and L and a positive scalar $T > 0$, suppose that there exist symmetric positive definite matrices P_1, P_3, Q_ε, Q_δ and a matrix P_2 of appropriate dimensions such that the matrix inequalities

$$\Phi_1 = \begin{bmatrix} & \begin{bmatrix} I & 0 \\ 0 & C_p' \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\ M & \\ * & -Q_\varepsilon \end{bmatrix} < 0, \qquad (15)$$

$$\Phi_2 = \begin{bmatrix} -\begin{bmatrix} P_1 & P_2 \\ P_2' & P_3 \end{bmatrix} & \begin{bmatrix} -K'R'S \\ 0 \end{bmatrix} & \Lambda_1(T)'\begin{bmatrix} P_1 & P_2 \\ P_2' & P_3 \end{bmatrix} \\ * & -2S & \Lambda_2(T)'\begin{bmatrix} P_1 & P_2 \\ P_2' & P_3 \end{bmatrix} \\ * & * & -\begin{bmatrix} P_1 & P_2 \\ P_2' & P_3 \end{bmatrix} \end{bmatrix} < 0, \qquad (16)$$

are verified with matrix M is given in (8) and

$$\Lambda_1(T) = e^{\left(\begin{bmatrix} A_p & -LC_p \\ 0 & A_p + LC_p \end{bmatrix} T\right)} + \int_0^T e^{\left(\begin{bmatrix} A_p & -LC_p \\ 0 & A_p + LC_p \end{bmatrix} s\right)} ds \begin{bmatrix} B_p K & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Lambda_2(T) = \int_0^T e^{\left(\begin{bmatrix} A_p & -LC_p \\ 0 & A_p + LC_p \end{bmatrix} s\right)} ds \begin{bmatrix} B_{pf} \\ 0 \end{bmatrix}. \qquad (17)$$

Then, the event-triggered sampling rule defined by (11) with

$$g\left(\delta(t), \begin{bmatrix} \hat{x}(t) \\ e_y(t) \end{bmatrix}\right) = \delta(t)'Q_\delta\delta(t) - \begin{bmatrix} \hat{x}(t) \\ e_y(t) \end{bmatrix}' Q_\varepsilon^{-1} \begin{bmatrix} \hat{x}(t) \\ e_y(t) \end{bmatrix} \qquad (18)$$

is such that the origin of system (7) is globally asymptotically stable. Furthermore, the inter-sampling times are lower bounded by T.

Proof. As for Theorem 1, the proof was removed from reason of place but can be obtained from the authors. □

Remark 3. Note that Zeno behaviors are avoided from the definition of the event-triggered rule (11) because the inter-sampling times are lower bounded by the positive scalar T. Actually, T represents a dwell-time implying that there is a minimum time T between two updates.

4. OPTIMIZATION ISSUES

Let us point out that, differently from Tallapragada and Chopra (2012), in Theorem 2 the inter-sampling times are directly obtained via the satisfaction of (16) without the need of additional *a posteriori* calculations. Furthermore, if the condition (15) holds then it is always possible to find a small enough T such that conditions (16) is verified. The parameter T appears then as a tuning parameter of the event-trigger problem. If T is too large, the conditions may be not feasible. Since a large T can lead to a performance degradation with respect to the

$$M = \begin{bmatrix} He\{P_1(A_p+B_pK)\} & (A_p+B_pK)'P_2 - P_1LC_p + P_2(A_p+LC_p) & P_1B_p & P_1B_{pf} - K'R'S \\ * & He\{P_3(A_p+LC_p) - P_2'LC_p\} & P_2'B_p & P_2'B_{pf} \\ * & * & -Q_\delta & -R'S \\ * & * & * & -2S \end{bmatrix}. \tag{8}$$

continuous-time implementation, a classical trade-off has to be considered when choosing T. Actually, it is important to note that provided that matrices A_p+B_pK and A_p+LC_p are Hurwitz, the existence of a small enough positive scalar T such that the conditions of Theorem 2 are feasible is ensured.

Conditions in Theorem 2 are LMIs provided that K, L and T are fixed, as typically considered in an emulation context. In order to select gains K and L, classical design techniques may be used, possibly considering some performance criteria. In particular, the observer gain L is chosen to get $A_p + LC_p$ Hurwitz with fast enough eigenvalues.

Hence, the selection of the parameters Q_ε and Q_δ can be systematically performed through the following convex optimization problem (given K, L and $T > 0$):

$$\min_{P_1,P_2,P_3,Q_\varepsilon,Q_\delta,S} trace(Q_\delta) + trace(Q_\varepsilon).$$

$$\text{s.t. } (15), (16). \tag{19}$$

The idea behind the optimization criterion in (19) is to get Q_δ and Q_ε as "small" as possible. With the view of the event triggering rule (18), and condition (15), this also means that the matrix $Q_\delta > 0$ is minimized as well as the matrix $Q_\varepsilon^{-1} > 0$ is maximized. Since the control input is updated whenever the function g in (18) is positive, this optimization procedure aims at reducing the impact of the first positive contribution in g ($Q_\delta > 0$) over the second negative contribution ($-Q_\varepsilon < 0$).

Let us emphasize that the dwell-time T is also a design parameter and its role is connected to the expected average sampling rate of the event triggered implementation. It can then be interesting to seek for maximizing the value of T through problem (19) by iteratively increasing T and testing LMI conditions. The implicit goal of such a convex optimization problem proposed is to reduce as much as possible the occurrences of sampling. This aspect is illustrated in Section 5.

5. NUMERICAL EXAMPLE

5.1 System description

Let us consider the following plant:

$$\begin{cases} \dot{x}_p(t) = \begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix} x_p(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} q(u(t)), \\ y_p(t) = [1\ 0]x_p(t), \end{cases} \tag{20}$$

where $q(u)$ is a logarithmic quantization function defined as follows (Fu and Xie (2005)):

$$q(u) = \begin{cases} \dfrac{1}{\rho_q^j} & \text{if} \quad \dfrac{1}{\rho_q^j(1+\eta_q)} \leq u < \dfrac{1}{\rho_q^j(1-\eta_q)}, \ j \in \mathbb{Z} \\ \\ -q(-u) & \text{if} \quad u < 0 \end{cases}$$

with

$$\eta_q = \frac{1-\rho_q}{1+\rho_q}, \quad 0 < \rho_q < 1$$

It should be noticed that the quantization error $\tilde{q}(u) = q(u) - u$ is restricted to the cone defined by $\pm\eta_q u$ (i.e., it satisfies the

T	Average number of control updates
0.1	15.38
0.2	15.48
0.3	15.36
0.4	14.54
0.5	13.45
0.6	13.82
0.7	14.68

Table 1. Average number of control updates for system (20) with the event-triggered rule (10) obtained by Theorem 2 for various values of T.

relation $(\tilde{q}(u)+\eta_q u)(\tilde{q}(u)-\eta_q u) < 0$ (de Souza et al. (2010)). In order to cast the system in the form (1), with a function $f(u)$ satisfying (2), it suffices to consider:

$$f(u) = \tilde{q}(u) - \eta_q u;$$

$$A_p = \begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix}; \quad B_{pf} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad B_p = (1+\eta_q)B_{pf};$$

$$C_p = [1\ 0]; \quad D_{pf} = 0.$$

Note that in this case the relation $(\tilde{q}(u)+\eta_q u)(\tilde{q}(u)-\eta_q u) < 0$ becomes $f(u)(f(u)+2\eta_q u) < 0$, i.e., (2) is verified with $R = 2\eta_q$.

5.2 Optimization results

We consider the quantization with $\rho_q = 0.9$ (which leads to $R = 2\eta_q = 0.105263$), the state feedback gain matrix $K = [-6\ -3]$, the observer gain matrix $L = [-3.5\ -7]'$ and the dwell-time $T = 0.2$. Then, solving the optimization problem (19) with this data, we obtain:

$$Q_\varepsilon = \begin{bmatrix} 0.452633 & 1.10763\cdot10^{-5} & -2.19703\cdot10^{-9} \\ 1.10763\cdot10^{-5} & 0.275705 & 1.82799\cdot10^{-10} \\ -2.19703\cdot10^{-9} & 1.82799\cdot10^{-10} & 1.51132\cdot10^{-5} \end{bmatrix},$$

$$Q_\delta = 0.728379.$$

5.3 Influence of T

Now, to show the influence of the parameter T, we solve optimization problem (19) for some values of T in the interval $[0.1, 0.7]$. Table 1 shows the average number of updates of the control signal obtained for each T chosen, considering 100 different initial conditions and $t \in [0, 10]$. The variation on the number of updates is approximately 12%. A value of $T \geq 0.8$ renders optimization problem (19) unfeasible.

5.4 Some simulations

We consider several values of T to depict the evolution of the plant and observer states, of the control input and of the

a) $T = 0.2s, \quad N_u = 11.$ b) $T = 0.4s, \quad N_u = 10.$ c) $T = 0.7, \quad N_u = 10.$
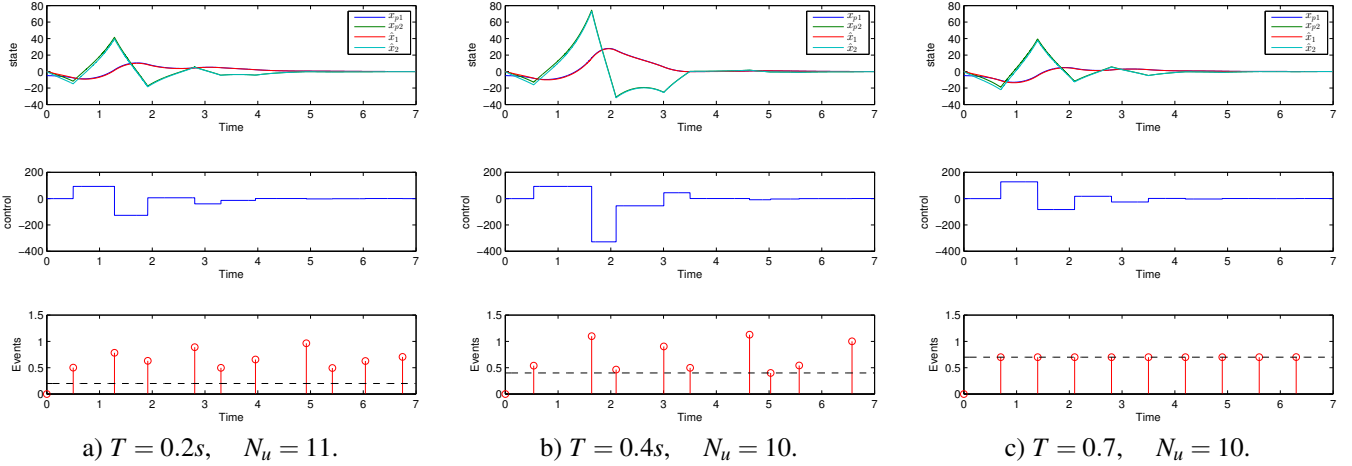
Fig. 1. Figure representing the state of the plant and the observer, the control input u, and the sampling instants issued from Theorem 2 with $T = 0.2$ (left), $T = 0.4$ (middle) and $T = 0.7$ (right).

sampling instants with the event-triggering rule (10) obtained by Theorem 2. To do this we consider the following initial conditions for the plant and the observer:

$$x_p(0) = \begin{bmatrix} -5 \\ 0 \end{bmatrix} \quad ; \quad \hat{x}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{21}$$

Fig. 1a) shows three simulations considering these values for the trigger function and initial conditions defined as in (21). The top plots depicts the plant and observer states. It can be noticed that the observer states quickly converge to the plant states, as expected. The control signal is shown in the middle plot. Note that the control signal is effectively held constant between two events. Moreover, the value of the control is subject to a logarithmic quantization. The bottom plot depicts the sampling instants, with the sizes of the bars representing the inter-event times, i.e. the difference between the time of that event and of the previous one. The dashed line marks the dwell-time. One can see that the trigger strategy is effectively delaying the sampling events, yielding inter-sampling times larger than the dwell-time while ensuring the asymptotic stability of the closed-loop system.

Figs. 1b) and 1c) show simulations for $T = 0.4$ and $T = 0.7$, respectively, both considering initial conditions defined as in (21). One can notice that, as T increases, more the inter-sample times are equal to the value of T, i.e. the event-triggered strategy cannot delay the sampling events so effectively anymore. For $T = 0.7$ and the initial condition considered, all the inter-sample times are equal to T. As mentioned before, this value of T is close to the limit of feasibility of optimization problem (19).

6. CONCLUSION

The paper handled an observer-based event-triggered control strategy for linear systems subject to cone-bounded nonlinearities affecting the input. The developed technique used only available variables. Sufficient conditions based on linear matrix inequalities (LMI) associated to convex optimization problem have been proposed to ensure the asymptotic stability of the closed loop in an emulation context. Moreover, the proposed approach allowed to design the event-triggering rule with a parameter T imposing the inter-sampling times.

These preliminary results let some open issues. In particular, it could be interesting to study the co-design problem, that is the simultaneous design of the control gains (K and L) and the triggering rule (i.e. matrices Q_δ and Q_ε). Furthermore, it could be pertinent to study the presence of cone-bounded nonlinearities depending on the state, imposing to change the route used to derive the conditions.

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