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To cite this version:
Marco Tognon, Antonio Franchi. Position Tracking Control for an Aerial Robot Passively Tethered to an Independently Moving Platform. 20th IFAC World Congress, Jul 2017, Toulouse, France. Proceedings of the 20th IFAC World Congress, 7p., 2017. <hal-01501919>

HAL Id: hal-01501919
https://hal.laas.fr/hal-01501919
Submitted on 4 Apr 2017

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Position Tracking Control for an Aerial Robot Passively Tethered to an Independently Moving Platform

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Abstract: We study the control problem of an aerial vehicle moving in the 3D space and connected to an independently moving platform through a physical link (e.g., a cable, a chain or a rope). The link is attached to the moving platform by means of a passive winch. The latter differs from an active winch by producing only a constant uncontrollable torque. We solve the problem of exact tracking of the 3D position of the aerial vehicle, either absolute or with respect to the moving platform, while the platform is independently moving. We prove two intrinsic properties of the system, namely, the dynamic feedback linearizability and the differential flatness with respect to the output of interest. Exploiting this properties we design a nonlinear controller able to exponentially steer the position of the aerial robot along any sufficiently smooth time-varying trajectory. The proposed method is tested through numerical simulations in several non-ideal cases.

Keywords: Aerial robot control, tethered aerial vehicles, nonlinear control

1. INTRODUCTION

The interest for aerial robots, sometimes called UAVs, UASs, or MAVs, saw a fast improving in the last few years. Indeed, thanks to light-weight and low-cost on-board sensors and computation units, these aerial vehicles became accessible for many companies and robotic labs. A new emerging trend is aerial manipulation in which aerial robots have to physically interact with the environment (Gioioso et al., 2014b; Yüksel et al., 2014; Gioioso et al., 2014a). Nevertheless, some of their typical limitations, such as limited flight time and maximum payload, seriously affect their performances. In particular they could be inapplicable for tasks requiring long time data acquisition or for inspection in the presence of external disturbances like strong wind.

In order to overcome these limitations the tethered solution has been already proposed by several works. In fact the physical link connecting the robot to the ground can be used to provide a constant power supply and a high-bandwidth communication channel, thus improving the flight time and the data transmission. For this system, controllers for the tracking/stabilization of the 2D vertical position of the vehicle while keeping the tether taut have been already presented (Tognon and Franchi, 2017; Lupashin and D’Andrea, 2013; Nicotra et al., 2014). Furthermore, in some of them (Tognon and Franchi, 2017; Lupashin and D’Andrea, 2013) it has been shown that, thanks to the link, an onboard inertial sensor is enough to retrieve the full state of the system wherewith to close the control loop, avoiding the use of GPS or onboard vision.

It has been also proved that the use of the link can improve the flight stability during risky maneuvers as take-off and landing from a moving platform (Sandino et al., 2014) and a strongly sloped surface (Tognon et al., 2016b), or flight in the presence of strong wind. The use of an actuated winch in order to control the length of the cable was exploited also in (Tognon et al., 2016a) where the problem of controlling the 3D position of an aerial vehicle tethered to a moving platform was addressed. However the use of an actuated winch could be not always the best solution or not even feasible. Indeed it requires a torque controlled motor that increases the mechanics and the electronics of the system, as well as its cost and weight. Moreover, the additional actuation corresponds to an increased controller complexity and of the needed set of sensors.

On the other hand, in this work we analyze a simpler mechatronic system by considering a passive winch applying only a non-controllable (constant) torque, e.g., generated by a spring-based mechanism. This in turn lets reduce the load of the winch itself that could in principle be easily handled by a human being, e.g., in a leash-like configuration. The only controllable actuation is then given by the orientable thrust generated by the underactuated aerial vehicle. In previous works like (Oh et al., 2006; Sandino et al., 2014), where they consider a stabilization problem only for the landing task. Differently in this manuscript we solve the more general exact tracking problem of a desired trajectory for the relative aerial robot position with respect to the platform. To achieve the control objective we prove some intrinsic properties of the system such as exact linearizability through dynamic feedback and differential flatness. Exploiting the first property, we design a nonlinear controller able to exponentially steer the output along the desired trajectory. The second property

* This work has been partially funded by the European Union’s Horizon 2020 research and innovation programme under grant agreement No 644271 AEROARMS.
In this work, the superscript indicates the frame of references.

Preprint version, 22nd IFAC World Congress, Toulouse, France (2017)
\[ Mq + c + g + n + w = Qu \] (3)

where \( M(q) \in \mathbb{R}^{3 \times 3} \) is the positive definite inertia matrix, \( c(q, \dot{q}, \omega_C) \in \mathbb{R}^3 \) contains all the centrifugal/Coriolis terms, \( g(q, R_C) \in \mathbb{R}^3 \) contains all the gravity terms, \( n(q, \dot{q}, \omega_C) \in \mathbb{R}^3 \) contains the terms depending on the acceleration of the moving platform, \( w(\tau_R) \in \mathbb{R}^3 \) contains the terms depending on the constant torque winch, and \( Q(q, R_R, R_C) \in \mathbb{R}^{3 \times 4} \) is related to the generalized forces, referred as \( a_u \), performing work on \( q \), such that:

\[ a_u = Q(q, R_R, R_C)u = -J_R^T R_C^T R_R e_3 \ 0_{3 \times 3}u, \]

where \( J_R = \frac{\partial \phi_R}{\partial q} \in \mathbb{R}^{3 \times 3} \) is the Jacobian matrix of \( \phi_R \), and \( u = [u_1 \ u_1'^T = [J_R \ \tau_R'^T]^T \in \mathbb{R}^4 \) is the input vector. For space limitation we do not report here the full expression of each term. The state of the system is then expressed by \( x = (q, \dot{q}, R_R, \omega_R) \). Finally, to complete the characterization of the system, we derive the link internal force from the balance of momenta on the winch along its longitudinal axis:

\[ f_L = J_W \dot{\tau}_W - \tau_W, \]

where \( J_W = J_W / \omega_W^2 \) and \( \tau_W = \tau_W / \omega_W^2 \).

The problems addressed in this work are the following.

**Problem 1.** Design a control law for the inputs \( f_R \) and \( \tau_R \) to let \( q \) (or equivalently \( \phi_R \)) thanks to inverse kinematics exactly track any sufficiently smooth desired trajectory.

**Problem 2.** Given the desired trajectory of \( q \), define a method to compute the nominal state and input in order to obtain the tracking of the desired trajectory.

The first objective allows to control the relative position of the aerial vehicle with respect to the moving platform, while the second can be exploited in a planning phase to design a desired trajectory that fulfills input limitations or that keeps the internal force bounded (see Sec. 4.1).

### 3. Dynamic Decoupling Control

Define as output of interest the quantity \( y_q = [y_1 \ y_2 \ y_3]^T = \dot{q} \in \mathbb{R}^3 \). Although the dimension of \( u \) is greater than the dimension of \( y_q \), the control problem is not redundant because \( \tau_R \) neither affects \( y_q \) nor its derivatives. Indeed \( \tau_R \) does not change the thrust vector direction, \( \tau_R \), that is the only quantity that, together with \( \tau_W \), plays a role in the dynamics of \( y_q \). To obtain a well posed tracking problem we have to complete the set of outputs with a quantity dynamically dependent on \( \tau_R \). For this purpose we can consider any generic parametrization \( \eta = [\eta_1 \ \eta_2 \ \eta_3]^T \in \mathbb{R}^3 \) of \( R_R \), such that \( \dot{\eta} = T_R \omega_R \), and \( \eta = T_R \eta \). Then, to obtain a well posed parametrization from (2) the dynamics of \( \eta \) is:

\[ \dot{\eta} = T_R \omega_R + T_R \eta J_R^{-1} (J_R \omega_R + \omega_R \times \omega_R) + [0_{3 \times 1} \ T_R J_R^{-1}]u = b_\eta(\eta, \dot{\eta}) + e_\eta(\eta)u. \]

Then we consider \( y_4 = \eta_1 \), where \( \eta_1 \) is any entry of \( \eta \), as additional angle that is dynamically dependent from \( \tau_R \), i.e., such that:

\[ \partial \eta_1 / \partial \tau_R = e^{T}_\eta T_R J_R^{-1} e_3 \neq 0. \]

Considering \( y = [y_q \ y_4]^T \) as output of interest and applying the feedback linearization technique, we need to differentiate each entry of \( y \) until the input appears. From equations (3) and (5), \( y \) has to be differentiated twice to see the input appear:

\[ \begin{bmatrix} \dot{y}_q \\ \dot{y}_4 \end{bmatrix} = \begin{bmatrix} M(N) \\ b_\eta \end{bmatrix} + \begin{bmatrix} \dot{M}(\eta) \\ e_\eta \end{bmatrix}u \]
\[ = \begin{bmatrix} M(N) \\ b_\eta \end{bmatrix} + \begin{bmatrix} -Q R_C^T R_R e_3 \\ 0_{3 \times 3} \end{bmatrix} + \begin{bmatrix} 0_{3 \times 3} \end{bmatrix} e^{T}_3 T_R J_R^{-1} \]
\[ = \dot{b}(x, X_C^0) + \dot{E}(x, X_C^0)u, \]

where \( M = M^{-1} \), \( b_\eta = e^{T}_\eta e_\eta \), \( e_\eta = e^{T}_3 \dot{E}_\eta \), \( a = -c - g - \dot{w} - w \), and \( x_C^0 = (x_C^{(i)}(i) - \omega^{(i-1)}(i)) \) for \( i = 1, 2, \ldots \). The decoupling matrix \( \dot{E} \) is singular for every condition because \( \tau_R \) does not appear on \( y_q \). This means that the system is not statically feedback linearizable.

In this case one can apply a dynamic feedback inserting a dynamic compensator in the control \( a_\eta \). Consider as new input the second derivative of the thrust and the torque, i.e., \( \dot{u}_1 = [u_1 \ u_1'^T \ u_1'']^T \). Now \( y_q \) has to be differentiated four times to see \( u \) appear, while for \( y_4 \) everything remains the same, indeed:

\[ \begin{bmatrix} \ddot{y}_q \\ \ddot{y}_4 \end{bmatrix} = \begin{bmatrix} \ddot{M}(\tilde{a}_\eta + a_u + \dot{J}_R^T R_C^T R_R \dot{e}_3 \ 0_{3 \times 1} \ end{bmatrix} \dot{\eta} \]
\[ \dot{\tilde{a}}_\eta + \ddot{a}_\eta = \ddot{a} \]

where \( \ddot{a} \) gathers all the terms that do not depend on \( \eta \).

Since \( J_R \) is diagonal, writing the skew symmetric matrix relative to \( J_R^{-1} \tau_R \) and doing some algebra we obtain:

\[ \begin{bmatrix} J_R^{-1} \tau_R \end{bmatrix} \times \dot{e}_3 = \left[ -\frac{e_2}{J_R^{(1)}} \frac{e_1}{J_R^{(2)}} \ 0_{3 \times 1} \right] \tau_R, \]

where \( J_{Rkn} \) corresponds to the element of the matrix \( J_R \) in the position \( k, m \). Replacing equations (10) and (9) into (8) we obtain:

\[ \begin{bmatrix} \ddot{y}_q \\ \ddot{y}_4 \end{bmatrix} = \dot{b}(x, X_C^0) + \begin{bmatrix} \ddot{M}(\tilde{a}_\eta + a_u + \dot{J}_R^T R_C^T R_R T \ 0_{3 \times 1} \ \dot{e}_3 \end{bmatrix} \dot{u}, \]

where \( x = (q, \dot{q}, R_R, \omega_R, f_R, \dot{f}_R) \) is the extended state, \( b(x, X_C^0) \) collects all the terms that do not depend on \( \dot{u} \), \( T = [e_3 - \frac{\omega^{(i-1)}(i)}{J_R^{(1)}} e_2 \ \frac{\omega^{(i-1)}(i)}{J_R^{(2)}} e_1] \), \( e_2 = e^{T}_3 T_R J_R^{-1} e_3 \), and \( e_3 = e^{T}_3 T_R J_R^{-1} \ 0_{3 \times 1} \ e_1 \ e_2 \). The decoupling matrix \( \dot{E} \) results to be invertible if \( \dot{e}_2 \neq 0 \) and if \( T \) is invertible, since \( R_R, R_C, J_q \) and \( M \) are always full rank (except where the model is singular, i.e., for \( l = 0 \) or \( \delta = \pm \pi / 2 \)). Thus it is easy to verify that \( \dot{E} \) is invertible if \( \dot{u}_1 \neq 0 \) and if the parametrization \( \eta \) of \( R_R \) and one of its elements \( \eta_1 \) are chosen such that \( \dot{e}_2 \neq 0 \), i.e., if (6) is verified in the domain of interest.

Then, in the case in which \( \dot{E} \) is invertible, defining \( v = [v_1 \ v_2 \ v_3 \ v_4]^T \) as virtual inputs, the control law:

\[ \dot{u} = \dot{E}(x, X_C^0)^{-1} \left[ -\dot{b}(x, X_C^0) + \dot{v} \right], \]

brings the original system in the equivalent linear system:

\[ \begin{cases} \dot{y}_1 = v_1, \\
\dot{y}_2 = v_2, \\
\dot{y}_3 = v_3, \\
\dot{y}_4 = v_4. \end{cases} \]

This means that the system results to be exactly linearizable through dynamic feedback and the linearized system (12) does not have an internal dynamics. Indeed,
To achieve the second objective we shall prove the differential flatness of the system with respect to the output $y$. This means that since the system is dynamic feedback linearizing output is also a flat output on an open and dense set of the state space (De Luca and Oriolo, 2002).

In other words we show that the state and the input of the system $\dot{x}$, and the inputs of the controller are only the state of the system, $f_R$ and $f_f$.

From the previous equation, in order to find the missing states, $R_R$ and $\omega_R$, and the inputs, $f_R$ and $\tau_R$, in function of $y$ and its derivatives, one can use the same method in (Mellinger and Kumar, 2011). We do not report here the explicit derivation for space limitation.

**Proposition 2.** The model (1), (2) and (3), is differentially flat with respect to the flat outputs $\mathbf{y} = [l \, \varphi \, \delta \, \eta_i]^T$ where $\eta_i$ is the yaw angle of $R_R$. In other words the state and the inputs can be written as algebraic function of $\mathbf{y}$ and a finite number of its derivatives.

4. DIFFERENTIAL FLATNESS

To achieve the second objective we shall prove the differential flatness of the system with respect to the output $y$. In other words we show that the state and the input of the system can be written as algebraic function of the output and a finite number of its derivatives (Fliess et al., 1995).

Thus, given the desired trajectory of the output one can compute the nominal state and inputs needed to follow the desired trajectory. This property is useful for planning and optimization in order to generate feasible trajectories under input limitations or other constraints on the state.

It has been proved that an exactly dynamical feedback linearizing output is also a flat output on an open and dense set of the state space (De Luca and Oriolo, 2002). This means that since the system is dynamic feedback linearizable, it is also differential flat. Nevertheless, it is useful to provide the explicit expression of $x$ and $u$ as function of $y$ and its derivatives when $\eta_i$ is the yaw angle.

Part of the state is directly given by $y_q$, i.e., $q = y_q$, $\dot{q} = \dot{y_q}$. Only the part of the state related to the rotational dynamics and the inputs have still to be derived. From (4) we can write the internal force as function of $\dot{y}_q$:

$$f_L = J_W \ddot{\theta} - f_M = f_{CL}(\dot{y}_q).$$

We can also write $d_C$ and $J_q$ as function of the output, i.e., $d_C = \dot{d}_C(y_q)$ and $J_q = J_q(y_q)$. Then, if the linear velocity, the attitude of the platform and their time derivatives are known, we can write $\ddot{p}_R^{\omega}$ as function of $y_q$, $\ddot{y}_q$ and $\dot{y}_q$ as

$$\ddot{p}_R^{\omega} = R_CL_C(p_C^\circ + \Omega_C d_C(y_q) + J_q(y_q) y_q) + \ddot{p}_C^\circ + \Omega_C d_C(y_q) + J_q(y_q) y_q + J_q(y_q) y_q].$$

Writing the force balance equation at $O_R$ we can retrieve the thrust vector as function of $y_q$ and its derivatives, as

$$f_R = -m_R \ddot{p}_R^{W}(y_q, \ddot{y}_q, \dot{y}_q, \dot{X}_C^W - f_{CL}(\dot{y}_q)R_CL_Cd_C(y_q) - m_R \dot{\eta}_i = f_{\omega}(y_q, \ddot{y}_q, \dot{y}_q, \dot{X}_C^W).$$

From the previous equation, in order to find the missing states, $R_R$ and $\omega_R$, and the inputs, $f_R$ and $\tau_R$, in function of $y$ and its derivatives, one can use the same method in (Mellinger and Kumar, 2011). We do not report here the explicit derivation for space limitation.

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4.1 Discussion on Link Internal Force Regulation

Notice that, differently from previous related works such as (Tognon and Franchi; Tognon et al., 2016a), the internal force along the link is not part of the flat output for this system. This means that its value can not be directly controlled. On the contrary, from (14), it is a byproduct of the desired output trajectory, and in particular of the desired link length acceleration. Nevertheless, in order to keep the internal force always positive and within a desired bound $B_{FL} = [f_M, f_M]$ where $f_M \in \mathbb{R}_{>0}$, we can exploit the flatness of the system to design suitable desired trajectories of $y$. In particular, from equation (14) we have that $f_M \in B_{FL}$ if and only if $\dot{\bar{l}} \leq \dot{\bar{L}}$, where $\ddot{\bar{l}} = \dot{\bar{L}} = \dot{\tau}_W - \dot{\bar{L}}$.

Notice that the steady configuration, $\ddot{\bar{l}} = 0$, belongs to $B_{L}$ if and only if $\dot{\bar{l}} \leq 0$ that in turn means $-\dot{\tau}_W \leq \dot{\bar{L}}$.

In particular, if for $\dot{\bar{l}} = 0$ we want a particular internal force value $f_M^*$ in $B_{FL}$, we have to design the passive winch such that $\dot{\tau}_W = \dot{\bar{L}} - \dot{\bar{L}}$. Another parameter of the winch that can be optimized is it inertia $J_W$. Indeed it affects how $B_{FL}$ is mapped on $B_{L}$, e.g., if we make $J_W$ small enough, big variations of $\dot{\bar{l}}$ imply small variations of the internal force and thus an almost constant internal force, $f_{CL} \approx f_{CL}^*$. 

5. NUMERICAL VALIDATION

Through numerical simulation we shall validate our control method showing that the output of interest exponentially
converges to the desired trajectory in case of initial error, as well as it shows good tracking performances in several non-ideal conditions.

We consider an aerial vehicle of mass \( m_R = 1 \text{[kg]} \) and inertia \( \mathbf{J}_R = 0.25 \mathbf{I}_3 \text{[kg} \cdot \text{m}^2]\), where \( \mathbf{I}_3 \in \mathbb{R}^{3 \times 3} \) is the identity matrix. The winch has a constant radius \( r_W = 0.2 \text{[m]} \) and a constant inertia \( J_W = 0.15 \text{[kg} \cdot \text{m}^2]\). In order to obtain a steady state internal force \( f_l^* = 5 \text{[N]} \), we set the torque winch \( \tau_W = -1 \text{[N]} \) (see Sec. 4.1). Finally, in order to obtain a sufficiently fast exponentially tracking, we set the controller gains \( k_i \) and \( k_d \) such that the error dynamics \( \xi_1 \) and \( \xi_4 \) have poles in \((-0.5, -1, -1.5, -2)\) and \((-0.5, -1)\), respectively, for \( i = 1, 2, 3 \).

We design the platform motion and the aerial vehicle desired trajectory in order to simulate a patrol-like task of a delimited area. The platform simply travel the perimeter of the area of interest while the aerial vehicle, after the take-off maneuver, at time \( t_{arc} \) has to loiter above the platform. Then, starting from time \( t_{land} \) the aerial vehicle has to land on the platform. The trajectories of the systems are plotted in Fig. 5b. Notice that take-off and landing are performed while the platform is moving, making these standard maneuvers non trivial.

To validate the control method and to test its robustness we performed several simulations in different non-ideal conditions: a) initial tracking error, b) variation of the model parameters, c) noise on the measured state used for the feedback, d) non ideal motor modeled as a first order system characterized by a certain time constant.

Case a): we initialized the system with an initial tracking error of \( 10^{\circ} \) for the elevation, of \( 5^{\circ} \) for the azimuth and of 0.5[m] for the link length. Looking at Fig. 3 we can notice that after a transient, the controller steers the output of interest along the desired trajectory. Notice that the internal force along the link remains always positive and close to the desired steady state value \( f_l^* \). Furthermore it is exactly equal to \( f_l^* \) whenever \( \bar{I} \) is zero.

Case b): in a real implementation it is very unlikely to have a perfect knowledge of the model and its parameters. For this reason we tested the robustness of the control method with a variation of the 5\% on all the model’s parameters (see Fig. 4). Due to the mismatch between real and nominal model, the feedback linearization is not exact and the error does not go to zero. However it remains always bounded showing nicely degrading and sufficiently good tracking performances. With further simulations we noticed that the system remains stable showing acceptable tracking errors up to a parametric variation of 50\%, proving the robustness of the proposed method. Above the system becomes unstable.

Case c): we tested the robustness of the proposed method injecting Gaussian noise on the measured state used to close the control loop. The power of the noise has the same value of the one noticed in (Tognon et al., 2016a) out of an observer based on noisy sensors. From Fig. 5 one can see that the error does not converge to zero but remains always bounded showing good and practically viable tracking performances.

Case d): in this simulation we considered the thrust and the toque of the aerial vehicle generated by non-ideal motors modeled as a first order system characterized by a time constant of 0.2 [s]. The results displayed in Fig. 6 show a very small tracking error, validating the robustness of the control method to this additional non-ideality.

6. CONCLUSIONS

In this manuscript we faced the problem concerning an aerial vehicle moving in the 3D space, tethered to a passive winch rigidly attached to an independently moving platform. For this robotic system we went beyond the simple regulation problem proposing a nonlinear tracking controller able to let the aerial robot position follow any desired trajectory. To prove the robustness of the proposed method, we developed a simulation environment which allowed us to test the system on a large number of scenarios.

The validation of the method with real experiment in a plausible scenario is left for future works, e.g., with minimalistic and onboard sensor settings (Franchi et al., 2015).


