

Set-membership functional diagnosability through linear functional independence

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Introduction

Before a system is put into operation and eventually diagnosed, diagnosability analysis is an important stage. Diagnosability indeed guarantees that the sensed values delivered by the available instrumentation can be processed into an appropriate set of symptoms allowing to discriminate a reasonable set of faulty situations.

A fault is considered as an additional parameter that impacts the behavior of some components of the system. Its effects may be linear or non-linear. Functional diagnosability, introduced in [1], and extended to set-membership (SM) functional diagnosability in [2] was analyzed through parameter identifiability. In the proposed work, SM-functional diagnosability is assessed from the linear independence of *SM-functional fault signatures*, which results in a much more direct test.

The system is assumed to be represented by the following model :

$$\Gamma \begin{cases} \dot{x}(t) = g(x(t), u(t), f, p), & x(t_0) = x_0, \\ y(t) = h(x(t), u(t), f, p), \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ and $y(t) \in \mathbb{R}^m$ denote the state variables and the outputs respectively. $u(t)$ is the input vector. The function g is real

and analytic on an open set of \mathbb{R}^n . p denotes the parameter vector belonging to a connected set $P \in \mathcal{U}_p$, where \mathcal{U}_p is an open set of \mathbb{R}^p .

The fault vector $f \in \mathbb{R}^e$ belongs to a connected set $F \subseteq \mathcal{F}_{SYS}$, where \mathcal{F}_{SYS} is the exhaustive set defining the fault domain. $Y(P, F, u)$ denotes the set of outputs, solution of Γ with the input u , the parameter vector P and the fault vector F . Considering a connected set of faults $F \subseteq \mathcal{F}_{SYS}$, let us denote by F_j the vector derived from F by setting all the components equal to zero except the j th component. F_j , called a “bounded fault”, is a connected set describing a faulty situation characterized by the occurrence of a fault whose magnitude is assumed to belong to the bounded set F_j .

Definitions and role of analytical redundancy relations

Using elimination theory, some differential polynomials or analytical redundancy relations (ARR) linking system inputs, outputs and their derivatives can be obtained. The use of ARRs makes possible to detect, isolate, and estimate the characteristics of a fault acting on the system. Specific ARRs indexed by $i = 1, \dots, m$ can be obtained [1] :

$$\begin{aligned} w_i(y, u, f, p) &= m_{0,i}(y, u, p) - \sum_{k=1}^{n_i} \gamma_k^i(f, p) m_{k,i}(y, u) \\ &= w_{0,i}(y, u, p) - w_{1,i}(y, u, f, p) \end{aligned} \quad (2)$$

where $(\gamma_k^i(f, p))_{1 \leq k \leq n_i}$ are rational in f and p , $\gamma_v^i \neq \gamma_w^i$ ($v \neq w$) and $(m_{k,i}(y, u))_{1 \leq k \leq n_i}$ are differential polynomials with respect to y and u .

$w_{0,i}(y, u, p)$ is equal to $m_{0,i}(y, u, p)$, hence the first part of the polynomial does not contain components of f . It corresponds to the residual *computation form* whereas $w_{1,i}(y, u, f, p)$ is known as the residual *internal form*. Let us notice that (2) can as well be interpreted for bounded faults (vector F) and uncertain but bounded parameters (vector P).

One may be interested in distinguishing types of faults, independently of their magnitude. For instance, it may be important to detect a leakage on a pipe but the amount of derived flow may not be relevant. Hence the notion of *SM-functional diagnosability* that comes through the notion of *SM-functional signature*.

Definition 1. The SM-functional signature of a bounded fault F_j is a function $FSig_{SM}(F_j)$ which associates to F_j the interval vector:

$$(w_{1,i}(Y(P, u), u, F_j, P))_{i=1,\dots,m}.$$

$FSig_{SM}^{(i)}(F_j)$ denotes the i th component of $FSig_{SM}(F_j)$ and corresponds to an interval function. $FSig_{SM}^{(i)}(F_j)$ consists in a set of trajectories generated in the presence of F_j and can be viewed as a tube of trajectories on the time interval $[t_0, T]$.

To distinguish the two tubes of trajectories generated by two different bounded faults, we propose the following definitions. The first one refers to *weak* SM-functional diagnosability and permits an intersection of the two tubes on a time subinterval whereas the second one refers to *strong* SM-functional diagnosability and requires the two tubes to be totally disjoint on $[t_0, T]$.

Definition 2. Two bounded faults F_j and F_k are SM-functionally discriminable if $FSig_{SM}(F_j)$ and $FSig_{SM}(F_k)$ are distinct, which includes the two following cases:

- there exists at least one index $i^* \in \{1, \dots, m\}$ and a time interval $[t_1, t_2] \subseteq [t_0, T]$ such that for all $t \in [t_1, t_2]$, $FSig_{SM}^{(i^*)}(F_1) \cap FSig_{SM}^{(i^*)}(F_2) \neq \emptyset$ and $FSig_{SM}^{(i^*)}(F_1) \not\subseteq FSig_{SM}^{(i^*)}(F_2)$ or $FSig_{SM}^{(i^*)}(F_2) \not\subseteq FSig_{SM}^{(i^*)}(F_1)$, in which case F_j and F_k are said to be **weakly SM-functionally discriminable**.
- there exists an index $i^* \in \{1, \dots, m\}$ and a time interval $[t_1, t_2]$ such that for all $t \in [t_1, t_2]$, $FSig_{SM}^{(i^*)}(F_1) \cap FSig_{SM}^{(i^*)}(F_2) = \emptyset$, in which case F_j and F_k are said to be **strongly SM-functionally discriminable**.

Definition 3. The model Γ given by (1) is weakly (resp. strongly) SM-functionally diagnosable for \mathcal{F}_{SYS} if any two bounded faults $F_j, F_k \subseteq \mathcal{F}_{SYS}$ are weakly (resp. strongly) SM-functionally discriminable.

Analysis of SM-functional diagnosability

To analyse SM-functional diagnosability, a criterion testing the linear independence of SM-functional fault signatures is proposed.

Definition 4. Rewriting $FS_{i,j} := FSig_{SM}^{(i)}(F_j)$, the SM-signature matrix is defined as $MSig = (FS_{i,j})_{1 \leq i \leq m, 1 \leq j \leq e}$. Define also the extended following matrix for the case $i = 1$:

$$W_{MSig} = \begin{pmatrix} FS_{1,1} & FS_{1,2} & \dots & FS_{1,e} \\ FS'_{1,1} & FS'_{1,2} & \dots & FS'_{1,e} \\ \vdots & \vdots & \vdots & \vdots \\ FS_{1,1}^{(e-1)} & FS_{1,2}^{(e-1)} & \dots & FS_{1,e}^{(e-1)} \end{pmatrix}. \quad (3)$$

- Proposition 1.** 1. Consider the case $i = 1$ – If $\det(W_{MSig}) \neq 0$, then the faults of the set $\mathcal{F}_{SYS} = \{F_1, \dots, F_e\}$ are discriminable, as well as any subset of faults within this set¹.
2. Consider the case $i = m > 1$ and assume that $m \leq e$ – If the minor $M_{I,J}$ of order α , $2 \leq \alpha \leq m$, of $MSig$ is not zero then the faults in the set $\{F_j, j \in J\} \subset \mathcal{F}_{SYS}$ are discriminable, as well as any subset of faults within this set.

Note that Proposition 1 gives conditions on the rank of the interval matrices. Proofs are omitted due to lack of space.

Conclusion

A new approach to test the concept of SM-functional diagnosability is proposed. It leads to a test based on the rank of an interval matrix formed from the SM-functional signatures of the faults.

References

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¹Note that the determinant of W_{MSig} is the classical wronskian.