Omnidirectional Aerial Vehicles with Unidirectional Thrusters: Analysis, Optimal Design, and Motion Control

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To cite this version:

HAL Id: hal-01704054
https://hal.laas.fr/hal-01704054
Submitted on 8 Feb 2018

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Abstract—This paper presents a theoretical study on omni-
directional aerial vehicles with body-frame fixed unidirectional
thrusters. Omnipus multi-rotor designs are defined as the ones
that allow to exert a total wrench in any direction using positive-
only lift force and drag moment (i.e., positive rotational speed)
for each rotor blade. Algebraic conditions for a design to be
omnipus are derived, a simple necessary condition being the
fact that at least seven propellers have to be used. An energy
optimal design strategy is then defined as the one minimizing
the maximum norm of the input set needed to span a certain
wrench ellipsoid for the adopted input allocation strategy. Two
corresponding major design criteria are then introduced: firstly,
a minimum allocation-matrix condition number aims at an
equal sharing of the effort needed to generate wrenches in any
direction; secondly, imposing a balanced design guarantees an
equal sharing of the extra effort needed to keep the input in
the non-negative orthant. We propose a numerical algorithm to
solve such optimal design problem and a control algorithm to
control any omnidirectional platform. The work is concluded with
informative simulation results in non-ideal conditions.

I. INTRODUCTION

Aerial vehicles have been thoroughly studied and applied
in several fields and for several tasks, from simple remote
sensing to the more challenging physical interaction with the
environment and humans. The latter have been firstly targeted
using unidirectional-thrust vehicles actuated by multiple
collinear rotors and endowed with cables [1], rigid tools [2],
[3] or more complex robotic arms [4]–[6]. These vehicles are
energy efficient but underactuated because of the unidirectional
of the total thrust in the body frame. Therefore i) the
vehicle orientation is coupled with its translational motion,
and ii) the system cannot instantaneously react to forces
with any direction. Recent solutions to these issues consist in
using multidirectional-thrust vehicles that can generate a
force in multiple directions and can control both position and
orientation independently. Examples are the platforms with
tilted unidirectional-thrust rotors (i.e., propellers generating
lift in only one direction), see, e.g., [7] and [8]. However, in
these platforms the set of feasible forces does not span any
direction in \( \mathbb{R}^3 \).

A special case is made by omnidirectional-thrust vehicles,
that can produce a force in any direction in the body frame.
This sub-class of vehicles is the most preferable, especially
for physical interaction, because it can be oriented in any
direction and can compensate/exert any force independently,
thus allowing applications that are impossible with other
platforms, including safe human interaction, 360° aerial
photography, etc.

In [9] and [10] two omnidirectional-thrust vehicles are
proposed with 6 and 8 tilted bidirectional-thrust rotors,
respectively. Such rotors are able to invert the direction
of the lift force by inverting either the motor rotation or
the propeller angle of attack. However such rotors have
several issues: i) scarceness of reversible Electronic Speed
Controllers (ESC) for brushless motors, ii) lower energetic
efficiency compared to unidirectional rotors, iii) lower con-
trollability of the exerted force at low speeds, and iv) ex-
tra mechanical complexity and increased weight and thus
energy consumption (in case of variable pitch propellers).
A solution to obtain an omnidirectional-thrust vehicle using
instead unidirectional-thrust rotors is to actively tilt the whole
propeller groups [11]–[13]. This also requires extra actuation
and weight, and cannot in general guarantee instantaneous
force exertion because of the non-negligible time the pro-
pellers need to re-orient themselves.

At the best of our knowledge, there are no works thor-
oughly investigating if and how it is instead possible to
obtain omnidirectional-thrust vehicles with fixed (non-tiling)
and uni-directional thrusters, a solution that would overcome
all the problems of the aforementioned solutions. An attempt
can be found in [14], where an ad-hoc optimization for an
hexarotor is performed using an additional thruster whose
position and orientation depend on the other six. The method
cannot be easily extended to generic multirotor platforms,
and the general theoretical problem still remains mostly
open.

Instead, in this work we provide the fundamental def-
ings, properties, and conditions needed to rigorously
address the problem in the general case of \( n \) propellers
having any arrangement. For example, it turns out that
an omnidirectional-thrust vehicle needs to have at least 7
fixedly attached unidirectional-thrust rotors. We propose an
algorithm computing the best (fixed) directions of the \( n \geq 7 \)
propellers that make the vehicle omnidirectional-thrust and
minimize the range of required control inputs to hover in
any orientation. Finally, we propose a full-pose controller
ensuring the input unidirectionality.

II. MULTIROTOR MODEL

We start by defining an inertial world frame \( \mathcal{F}_W = \{O_W, x_W, y_W, z_W\} \) where \( O_W \) is its origin, placed arbitrarily,
and \( (x_W, y_W, z_W) \) are the orthogonal unit vectors. We con-
sider \( z_W \) parallel and opposite to the gravity vector. Then we
define the body frame $\mathcal{F}_R = \{O_R, x_R, y_R, z_R\}$ rigidly attached to the vehicle and centered in $O_R$, the vehicle center of mass (CoM). Position of $O_R$ and orientation of $\mathcal{F}_R$ w.r.t. $\mathcal{F}_W$ are described by the vector $p_R \in \mathbb{R}^3$ and the rotation matrix $R_R \in SO(3)$, respectively. Then we define by the vector $v_R \in \mathbb{R}^3$ the translational velocity of $O_R$ expressed in $\mathcal{F}_W$, and by $\omega_R \in \mathbb{R}^3$ the angular velocity of $\mathcal{F}_R$ w.r.t. $\mathcal{F}_W$ and expressed in $\mathcal{F}_R$. The generic vehicle is depicted in Fig. 1.

The matrixes $R$ and moment of inertia about $0J$ have the following structure

\[
\begin{bmatrix}
    m_R & I_3 & \Phi_R \\
    0 & J_R & \Omega_R \\
    \Phi_R & \Omega_R & \Phi_R \\
\end{bmatrix}
\]

where $e_3 = [0 \ 0 \ 1]^T$, $\Omega_R = S(\omega_R)$ is the skew symmetric matrix relative to $\omega_R$, $f \in \mathbb{R}^3$ and $m \in \mathbb{R}^3$ are the controllable total input force and torque expressed in $\mathcal{F}_R$, respectively.

Considering a multirotor with $n$ rotors, each of them produces a lift force and a moment due to the drag force [15]. All together they generate the total force (or thrust) and moment, $f$ and $m$, respectively, expressed as:

\[
w = [f^T \ m^T]^T = [F_1^T F_2^T]^T [u_1 \ldots u_n]^T = Fu.
\]

The matrices $F \in \mathbb{R}^{6\times n}$, $F_1 \in \mathbb{R}^{3\times n}$, and $F_2 \in \mathbb{R}^{3\times n}$ are called the full allocation matrix, the force allocation matrix and the moment allocation matrix, respectively. The control $u_i \in \mathbb{R}$ is typically equal to $w_i |w_i|$, where $w_i \in \mathbb{R}$ is the $i$-th propeller rotational speed. $F_1$ and $F_2$ have the following structure

\[
F_1 = \begin{bmatrix} v_1 & \ldots & v_n \end{bmatrix},
\]

\[
F_2 = \begin{bmatrix} d_1 \times v_1 & \cdots & d_n \times v_n + c_1 k_1 v_1 & \cdots & c_n k_n v_n \end{bmatrix},
\]

where $v_i \in \mathbb{R}^3$ are the coordinates, in $\mathcal{F}_R$, of the lift force generated by the $i$-th propeller when $u_i = 1$. In this formulation the aerodynamic coefficient that maps propeller speed into thrust intensity, typically called lift factor $c_l$, is $c_l = |v_i|/v_i$; ii) $d_i$ is the position of the center of the $i$-th propeller in body frame; iii) $c_i = -1$ ($c_i = 1$) if the $i$-th propeller angular velocity vector has the same direction of $v_i$ ($-v_i$) when $u_i > 0$, i.e., the propeller spins CCW (CW) when watched from its top; iv) $k_i \in \mathbb{R}$ is the constant ratio between the $i$-th propeller lift force and the drag moment, typically denoted with $c_z c_f$ in the literature. In conclusion, we recall a well known fact.

**Fact 1** (translation invariance). $F$ does not change if $d_i$ is replaced with $d_i + \lambda_i v_i$ for any $i = 1, \ldots, n$ and $\lambda_1, \ldots, \lambda_n \in \mathbb{R}$.

### III. Omniplus Designs

We introduce now the basic concept of multirotor design. Let us first define $c = [c_1 \cdots c_n]^T$ and $k = [k_1 \cdots k_n]^T$.

**Definition 1.** A multirotor design $\mathcal{D}$ is a tuple $\mathcal{D} = (n, c, k, d_1, \ldots, d_n, v_1, \ldots, v_n)$, which describes the number of propellers $n$, their aerodynamic characteristics, locations and orientations w.r.t. $\mathcal{F}_R$. We call the tuples $(v_1, \ldots, v_n)$ and $(n, c, k, d_1, \ldots, d_n)$ the vectoring part and the ectoring part of $\mathcal{D}$, respectively.

We denote with $I$ the column vector with all ones. Its size is understood from the context. Given two vectors $x$ and $y$, the notations $x \geq y$, $x > y$ have to be intended component-wise.

**Definition 2.** Given $u \geq 0$, a multirotor design $\mathcal{D}$ is an $omniplus$ ($\mathcal{O}_+u$) if the corresponding full allocation matrix $F$, satisfies

\[
\forall w \in \mathbb{R}^6 \ \exists u \geq u^\dagger \ \text{s.t.} \ \ Fu = w.
\]

A design that is $\mathcal{O}_+u$ for any $u \in \mathbb{R}^6$ is said omniplus ($\mathcal{O}_+$). Considering $u \geq 0$ accounts for the possible presence of a minimum rotational speed constraint for the propellers.

**Proposition 1** (Theorem 1 in [16] extended to the $\mathbb{R}^6$ case). The following two conditions are equivalent

\[
\forall w \in \mathbb{R}^6 \ \exists u \geq 0 \ \text{s.t.} \ \ Fu = w, \quad \text{rank}(F) = 6 \quad \text{and} \quad \exists b > 0 \ \text{s.t.} \ \ Fb = 0.
\]

Proof. The same as in [16] but replacing 3-dimensional vectors with 6-dimensional vectors.

**Corollary 1.** Condition (4) is equivalent to (5) and (6), and as a consequence, any $\mathcal{O}_+u$ design is also $\mathcal{O}_+$.

Proof. Sufficiency is trivial. For necessity, consider a $u$ satisfying (5). Thanks to (6) consider $u' = u + \lambda b/\|b\|$ which satisfies both $u' \geq u^\dagger$ and $Fu' = w$ and therefore fulfills (4).

**Corollary 2.** For any design that is $\mathcal{O}_+$ it must be $n \geq 7$.

Proof. We have that it must be $\text{rank}(F) = 6$ and at the same time $0 \neq b \in \text{null}(F)$, therefore it must be at least $n = 7$. 

**Remark.** It is interesting to note that Prop. 1 and Corol. 2 find their counterpart in the literature of frictionless contact grasping (see [17] and the references therein).

**Proposition 2.** Let be given an $\mathcal{O}_+u$ design $\mathcal{D}$. Any design $\mathcal{D}'$ with the same ectoring part of $\mathcal{D}$ and a new vectoring part $(\alpha v_1, \alpha v_2, \ldots, \alpha v_n)$, where $\alpha \neq 0$, is also $\mathcal{O}_+u$.  

---

Fig. 1: Schematic representation of a multirotor and its main quantities. Only three of the $n$ propellers are shown.
Proof. Denote with $F$ and $F'$ the full allocation matrixes of $\mathcal{D}$ and $\mathcal{D}'$, respectively. We have that $F' = \alpha F$, therefore the properties (6) are valid also for $F'$ as long as $\alpha \neq 0$.

In the following we denote with $I_j$ the $j$-by-$j$ identity matrix. We also use the following notation $v = [v_1 \ldots v_n]$ and $d = [d_1^T \ldots d_n^T]^T$. We can then rewrite (2) and (3) as

$$F_1 = [I_1 v_1 \ldots I_n v_n]$$

$$F_2 = [(S(d_1) + c_1 k_1 I_1) v_1 \ldots (S(d_n) + c_n k_n I_3) v_n] .$$

Proposition 3. A multirotor design is $O_+$ if and only if

$$\text{rank}(F) = 6,$$  \hspace{1cm} (9)

and \( \exists \ b = [b_1 \ldots b_n]^T > 0 \) s.t.

$$b_1 I_3 \ldots b_n I_3 \left[ b_1 (S(d_1) + c_1 k_1 I_1) \ldots b_n (S(d_n) + c_n k_n I_3) \right] v = 0$$

$$A(c, k, d_1, \ldots, d_n, b)$$

Proof. The condition (9) is the first part of (6). The condition (10) is obtained from the second part of (6) by using (7) and (8) and imposing $Fb = 0$.

IV. ALLOCATION STRATEGIES FOR $O_+$ DESIGNS

In this section we introduce two different input allocation strategies for $O_+$ designs. The first one, defined as the solution of Prob. 1, is the optimal one but is hard to be exploited for an analytically sound optimization of the design (unless a completely numerical algorithm is used). The second one, defined as the solution of Prob. 2, is suboptimal, but is amenable of a clear geometrical interpretation which can be used for an analytically sound design optimization.

Problem 1. Consider a given $O_+$ design with full allocation matrix $F$. Given a desired $w \in \mathbb{R}^6$, with $w \neq 0$, find the input $u \in \mathbb{R}^n$ s.t. $Fu = w$, $u \succeq u_1$, and $|u|$ is minimized.

The solution to Prob. 1 without the constraint $u \succeq u_1$ is $u^* = F^T w$, where $F^T$ is the Moore-Penrose pseudo-inverse of $F$. However, for a $O_+$ design it is never $u^* \succeq 0$, a part from the trivial case $w = 0$, as stated next.

Proposition 4. Let $F$ be the full allocation matrix of an $O_+$ design and $F^T$ its Moore-Penrose pseudo-inverse. For any desired wrench $w \neq 0$, the minimum norm solution of $Fu = w$, i.e., $u^* = F^T w$, has always at least a negative entry, hence it is never a solution to Problem 1.

Proof. We have that $\text{im}(F^T) = \text{im}(F^T)$ [18] and that $\exists b \in \text{null}(F)$ s.t., $b > 0$. Since $\text{im}(F^T)$ is orthogonal to $\text{null}(F)$, we have that $b^T u^* = 0$. If $w \neq 0$ then $u^* \neq 0$, and since $b > 0$, $u^*$ must have at least a negative entry for $b^T u^* = 0$ to hold.

Prop. 4 implies that the solution of Prob. 1 is always of the form $u = u^* + y$ with $y \in \text{null}(F)$. In particular, exploiting the fact that $u^* \perp y$, the solution structure is $u^* = u^* + y^*$, where

$$y^* = \arg \min_{y \succeq 0, F^T y = 0} \| y \|,$$  \hspace{1cm} (11)

which can be efficiently solved with any constrained QP solver.

To provide a geometrical understanding of the structure of the solutions of the input allocation problems, let us consider an ellipsoid that may, e.g., represent the set of desired attainable wrenches $S_w = \{ w \in \mathbb{R}^6 | w^T \Sigma w \leq 1 \} \subset \mathbb{R}^6$, where $\Sigma \in \mathbb{R}^{6 \times 6}$ is a positive definite matrix. The ellipsoid $S_w$ is mapped by $F^T$ to the set $E_w^* = \{ u \in \mathbb{R}^n | u = F^T w, \forall w \in S_w \} \subset \mathbb{R}^n$ – an idealized representation from $\mathbb{R}^2$ to $\mathbb{R}^3$, and with $\Sigma = I$ is shown in Fig. 2. The set $E_w^*$ is a 6-dimensional ellipsoid of $\mathbb{R}^n$, contained in the subspace $\text{im}(F^T)$, whose shape is defined by the singular value decomposition of $F$ and $\Sigma$. There is a one to one correspondence between each $w \in S_w$ and each $u \in E_w^*$. However, according to Prop. 4, any vector $u \in E_w^*$ has always at least a negative entry (a part from $u = 0$). In order to satisfy the constraint $u \succeq u_1$ one has to project each point $u^*$ of $E_w^*$ onto one of the external facets of the shifted non-negative orthant denoted from now on with $\mathbb{R}^n_+$. The projection must be done by adding to $u^*$ a perpendicular vector that belongs to $\text{null}(F)$ and has minimum norm, i.e., obtaining $y^*$ by solving (11). By doing so for all the points in $E_w^*$ we obtain the set of solutions of Prob. 1 defined as $E_w^u = \{ u^* + y^* \in \mathbb{R}^n | u^* \in E_w^*, \text{ and } y^* \text{ solves (11)} \}$.

Denote with $\mathbb{R}^n_+$ the positive orthant of $\mathbb{R}^n$ and let us consider the following alternative Problem.

Problem 2. Consider a given $O_+$ design with full allocation matrix $F$ and let be given a constant vector $b \in \text{null}(F) \cap \mathbb{R}^n_+$. For any desired $w \in \mathbb{R}^6$, with $w \neq 0$, find the input $u = u^* + \lambda b \in \mathbb{R}^n$, where $\lambda > 0$, s.t., $u \succeq u_1$, and $|u|$ is minimized.

Problem 2 represents a restriction of Prob. 1 in the sense that a solution of Problem 2 satisfies the constraints of Prob. 1 but is in general sub-optimal, since the solutions are searched only of the form $u = u^* + \lambda b$ where $b$ is a fixed
vector in null($\mathbf{F}) \cap \mathbb{R}^{d}_{++}$ (which always exists, thanks to (6)), and $\lambda > 0$ is a large enough positive scalar that ensures that each entry of $\mathbf{u}$ is not smaller than $\mathbf{u}$. Since it is structurally $\mathbf{u}^\top \lambda \mathbf{b}$, in order to minimize the norm of $\mathbf{u}^\top + \lambda \mathbf{b}$, one has to choose
\[
\lambda = \lambda^\circ (\mathbf{u}^\top, \mathbf{b}, \mathbf{u}) = \min_{\mu | \mathbf{u}^\top + \mu \mathbf{b} \geq 2} \mu,
\]
thus obtaining
\[
\mathbf{u}^\star := \mathbf{u}^\top + \lambda^\circ \mathbf{b}.
\]
By doing so we are projecting the set $E_u^\star$ on the facets of $\mathbb{R}^{d_{ij}}$ following the constant direction defined by $\mathbf{b}$. We denote this projection with $E_u^\star$ (e.g., $E = E_{u,\star}$ for $\mathbf{u}^\top$ or $E = E_{u,\star}^2$ for $\mathbf{u}^\top$).

Even if minimization (11) can be efficiently solved with any constrained QP solver, there is, at the best of our knowledge, no analytical form to express $\mathbf{y}^\top$. Furthermore, $\mathbf{y}^\top$ may in general change (in both norm and direction) depending on the particular $\mathbf{u}^\top \in E_u^\star$. This makes hard to understand how is the shape of $E_u^\star$ and, especially, how the value of $\max_{\mathbf{u} \in E_u^\star} ||\mathbf{u}||$ are influenced by the changes of the design parameters. Hence, developing an analytically sound design optimization around the first allocation policy is left as future investigation.

The second allocation strategy is instead amenable of a more clear geometrical interpretation that leads naturally to the following optimization problem.

**Problem 3.** Let be given an etero-vectoring part $(n \geq 7, c, k, d_1, \ldots, d_n)$. Find a vectoring part $(v_1, \ldots, v_n)$ that solves
\[
\min \text{cond}(\Sigma^{-1}\mathbf{F})
\]
subject to
\[
\mathbf{v}^\top D_i \mathbf{v} = v, \quad \mathbf{v}^\top D_i \mathbf{v} = v
\]
\[
\text{rank}(\mathbf{F}(c, k, d_1, \ldots, d_n, v)) = 6
\]
\[
A(c, k, d_1, \ldots, d_n) \mathbf{v} = 0,
\]
where $D_i = \text{diag}(D_{i1}, \ldots, D_{in})$ is a $3n$-by-$3n$ diagonal matrix in which $D_{ij}$, for $j = 1 \ldots n$, are 3-by-3 matrices such that $D_{ij} = 0$ for $j \neq i$ and $D_{ii} = I_3$.

**A. On the Existence of Solutions**

Determining which are the conditions on the etero-vectoring part that ensure the existence of a solution for Problem 3, and how to analytically compute a solution $\mathbf{v}$, are both still open questions which are left as future work. In the following we shall assume that a solution is computed in a numerical fashion. We empirically noticed that it is not practically hard to find numerical solutions for an etero-vectoring part whose parameters are chosen following the next common sense rules.

Firstly, the vectors $d_1, \ldots, d_n$ are chosen coplanar and in a star-shaped configuration, i.e., selecting any $d_i$ such that $d_i \times e_3 \neq 0$, and then choosing $d_i = R_c(2\pi(i-1)/n)d_i$ for $i = 2, \ldots, n$, where $R_c(\theta)$ is the canonical rotation matrix about the $z$-axis of an angle $\theta$. The coplanar constraint does not restrict the generality of the results. One could use any other etero-vectoring part. However, any 3D configuration of $\mathbf{d}$ can be reduced to a planar one. Indeed, once obtained the vectoring part, one can move the $i$-th thruster along the $\mathbf{v}_i$ direction exploiting Fact.1. This feature might be also exploited to avoid collisions between propellers and the main frame. The constraint $||d_i|| = ||d_i|| \forall i = 2 \ldots n$ is instead added for mechanical simplicity. Secondly, the vector $\mathbf{c}$ showing a balanced set of $-1$ and $1$ entries, e.g., $c_i = (-1)^i$ for $i = 1 \ldots n$. Thirdly, it is imposed $||v_i|| = v$ and $k_i = k \forall i = 1 \ldots n$, since it is common to use the same propellers in the same multirotor. Based on our experience, the algorithm described next has always been able to find a solution to Problem 3 with any etero-vectoring part of the class defined by the three rules above.

**B. Algorithm**

A simple but effective method to solve Prob. 3 is provided in Algorithm 1 and explained in the following.

First of all, $\text{randomVectoring}(n, v)$ generates the $v_i$’s, for $i = 1, \ldots, n$ as $v_i = v n_i$, where $n_i \in S_2 = \{n \in \mathbb{R}^3 || n || = 1\}$ are sampled randomly with a uniform probability. The resulting vectoring part fulfills only (15) among the constraints, and cannot be used as initial guess for a nonconvex numerical solver since constraint (17) is not yet satisfied. In order to find an initial guess that satisfies (17) we use an iterative algorithm which tries to find the solution to $\min_{\mathbf{v}} ||\mathbf{Av}||^2$ subject...
Algorithm 1: Optimal Omniplus Design

Input: ector-vectoring part, \( (n, c, k, d_1, \ldots, d_n) \)
Output: design \( \mathcal{D}^* = (n, c, k, d_1, \ldots, d_n, v_1, \ldots, v_n) \)

1. \( \mathcal{D}^* \leftarrow \emptyset \)
2. for \( k = 1 \) to \( N \) do
3. \( v^0 \leftarrow \text{randomVectoring}(n,v) \)
4. for \( j = 0 \) to \( N_k - 1 \) do
5. \( v^{j+1} \leftarrow v^j - A'v^j \) \% Newton-Raphson
6. \( v^{j+1} \leftarrow \text{renormalization}(v^{j+1}, v) \)
7. \( \mathcal{D}^* \leftarrow \text{locOptimumDesign}(n, c, k, d_1, \ldots, d_n, v^{N_k}_1, \ldots, v^{N_k}_N) \)
8. if \( \mathcal{D}^* = \emptyset \) or \( 0 < \text{condNum}(\mathcal{D}^*) < \text{condNum}(\mathcal{D}^*) \) then
9. \( \mathcal{D}^* \leftarrow \mathcal{D}^* \)
10. return \( \mathcal{D}^* \)

Fig. 3: Performances of the algorithm. Left: inner loop error for 10 different initial guesses. Right: cond(F) for 5 trials of the algorithm.

Fig. 4: Optimized omnibus design (cond(F) = 3.59) with 7 propellers. The red tick lines correspond to the vectors \( d_1, \ldots, d_7 \) arms all departing from \( O_R \). The black one defines \( d_7 \). The blue spheres correspond to the positions of the motors. The colored slim lines indicate the lift force direction of each propeller. The star and the square symbols indicate CCW and CW propellers, respectively.

VI. CONTROL STRATEGY

Given desired position and orientation trajectories, i.e., \( p_R^d(t) \) and \( R_R^d(t) \), respectively, the control strategy of a platform with an omnibus design is rather straightforward. In fact, one has to first decide the force and moment vector \( w^d \) to be applied to the body to steer the output along the desired trajectory. One can use a nonlinear model inversion combined with a Feed Forward plus a PID inner loop:

\[
\begin{align*}
\mathbf{w}^d &= G_R^{-1} \left( \mathbf{M}_R - \mathbf{b}_R \mathbf{a}^d + \mathbf{K}_D \mathbf{e} + \mathbf{K}_P \mathbf{e} + \mathbf{K}_I \int_0^t \mathbf{e} \, \mathrm{d}t \right), \\
\mathbf{e} &= (\mathbf{v}_R^d - \mathbf{v}_R)^	op \left( \mathbf{R}_R \mathbf{R}_R \mathbf{R}_R \mathbf{R}_R - \mathbf{I} \right), \\
\mathbf{e}_R &= [1/2 \mathbf{R}_R \mathbf{R}_R - \mathbf{R}_R \mathbf{R}_R] \mathbf{e}.
\end{align*}
\]

where \( \mathbf{K}_D, \mathbf{K}_P, \mathbf{K}_I \in \mathbb{R}^{6\times6} \) are positive diagonal definite matrices, \( \mathbf{e} = (\mathbf{v}_R^d - \mathbf{v}_R) \), \( \mathbf{e}_R = \frac{1}{2} \mathbf{R}_R \mathbf{R}_R - \mathbf{R}_R \mathbf{R}_R \mathbf{R}_R \mathbf{R}_R \frac{1}{2} \mathbf{e} \).

VII. NUMERICAL SIMULATIONS

In this section we shall present the simulation results validating the algorithm to find an optimal omnibus design and the proposed controller. We chose to use the minimum number of propellers, i.e., \( n = 7 \). Then we used an ector-vectoring part where \( d_i = 0.4 \mathbf{R}_R (2 \pi (i - 1)/n) [0 \, 0 \, 0] \) and \( k_i = 0.0192 \) [m] for \( i = 1, \ldots, n \). Furthermore, considering a standard motor-propeller with diameter 0.30 [m] available in the market, we have that \( v = 9.9 \cdot 10^{-4} \) [N/Hz] and \( y = 16^2 \) [Hz^2]. On the other hand, the maximum control input is equal to \( \mathbf{u} = 0.030 \) [H^2]. Mass and inertia of the vehicle are \( m_R = 1.3 \) [Kg] and \( J_R = \text{diag}(0.030, 0.030, 0.030) \) [Kg·m^2], respectively.

We finally completed the omnibus design running the proposed Algorithm 1. Figure 4 shows the design used in the following simulations for which it has been achieved cond(F) = 3.59. The optimized vectoring part is equal to:

\[
\begin{bmatrix}
v_1 & \cdots & v_n
\end{bmatrix} =
\begin{bmatrix}
-0.71 & 0.11 & 0.41 & 0.44 & 0.57 & -0.64 & -0.17 \\
0.67 & 0.04 & 0.85 & -0.35 & -0.38 & -0.58 & -0.26 \\
0.11 & -0.98 & 0.31 & 0.81 & -0.72 & -0.48 & 0.94
\end{bmatrix}
\]

In order to fully show the capability of the proposed design, we ask the vehicle to translate and rotate at the same time. The translational trajectory is a spline from the initial position to a desired final one. For the orientation, we planned a trajectory such that the z-axis of \( \mathcal{F}_R \) circles many times around the one-radius sphere. In this way we
show the conditions that have to be satisfied to obtain the omnidirectional-thrust property and propose an algorithm to generate such design in an optimal way. We also propose a nonlinear controller to track position and orientation trajectories demonstrating the lowest possible inputs for the optimized platform.

Based on those fundamental results many other works could sprout up from the community. An example could be the mentioned improvement of the optimization algorithm, perhaps exploiting the noticed symmetries on an optimized platform, and considering the thrust position as well. The formal proof that balanced design with \( \text{min} \text{cond}(F) \) minimizes the norm of the input is also left as future work, as well as the real implementation of such optimized platform.

**REFERENCES**


Additional Analysis and Simulations for an Omnidirectional-thrust vehicle with Only Fixed Unidirectional Thrusters

Technical report of:
“Omnidirectional Aerial Vehicles with Unidirectional Thrusters: Theory, Optimal Design, and Control”
IEEE Robotics and Automation Letters

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Abstract— This document is a technical attachment to [1] as an extension of the numerical validation part. Here we present additional simulations in presence of non-ideal conditions as noise, parameter variations, non-ideal motors, control input delays and external disturbances. A through validation of the robustness of the proposed method against the previously mentioned non-idealities is conducted.

I. HOW TO CITE THIS WORK
This technical report is accompanying our IEEE Robotics and Automation Letters paper [1]. If you wish to reference this work, please cite this paper as follows:


II. ADDITIONAL SIMULATION
In this section we present some additional simulations performed to validate the proposed method in ideal and non-ideal conditions.

A. Ideal Conditions
Figure 1 shows more detailed plots of the simulation done in ideal condition presented in Sec. VII of the paper.

B. Standard Non-ideal Conditions
We simulated the system in non-deal condition considering:
• Gaussian noise added to the state measurement with standard deviation equal to $\sigma_p = 0.01$ [m], $\sigma_v = 0.02$ [m/s], $\sigma_R = 3$ [°] and $\sigma_\omega = 0.1$ [rad/s] for the position, linear velocity, attitude and angular velocity, respectively. This corresponds to the standard deviation of standard pose estimators for aerial vehicles, in order to simulate real sensors;
• non-ideal motors modeled as a first order system with time constant equal to 0.08 [s];
• parameters uncertainty for mass and inertia matrix equal to 5% of the nominal value.

Fig. 2 shows the tracking performance and the inputs of the closed loop system between 40 [s] and 45 [s]. Outside of this interval the behavior is the same. One can notice that the tracking error is relatively small and, most importantly, the control inputs do not increase with respect to the ideal case.

C. Noise Robustness
Here we investigate the performances of the proposed method under increasing noise intensity. In particular, we performed several simulations in which the standard deviation of the noise varies from 0 to a maximum value of $\sigma_p = 0.01$ [m], $\sigma_v = 0.02$ [m/s], $\sigma_R = 3$ [°] and $\sigma_\omega = 0.1$ [rad/s], that corresponds to a very bad sensorial setup. This analysis shows how the tracking performance would get worse with the degradation of the sensorial set-up, e.g., moving from

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This work has been funded by the European Union’s Horizon 2020 research and innovation programme under grant agreement No 644271 AEROARMS.
motion capture system-like setup to a very poor gps. Notice that the chosen maximum standard deviations are higher than typical values obtained from standard state estimators, even using on-board sensors as vision or gps.

In order to show the results with an increasing noise, for each performed simulation we set $\sigma_\epsilon = \Delta \sigma \hat{\sigma}_\epsilon$, with $\Delta \sigma \in [0,1]$ for each noise.

For every value of $\Delta \sigma$, in Fig. 3, we show the mean value and the standard deviation of the norm of the tracking error in position and attitude, i.e., $\bar{\epsilon}_{pR}$, $\sigma_{\epsilon_{pR}}$, $\bar{\epsilon}_{R_\theta}$ and $\sigma_{\epsilon_{R_\theta}}$, respectively. Those quantities are computed as in the following:

$$\begin{align*}
\epsilon_{pR} &= ||p_R^e - p_R||^2 \\
\epsilon_{R_\theta} &= ||\epsilon_{R_\theta}||^2 \\
\bar{\epsilon}_{pR} &= \frac{1}{T} \int_0^T \epsilon_{pR}(t)dt \\
\sigma_{\epsilon_{pR}} &= \frac{1}{T} \int_0^T (\epsilon_{pR}(t) - \bar{\epsilon}_{pR})dt \\
\bar{\epsilon}_{R_\theta} &= \frac{1}{T} \int_0^T \epsilon_{R_\theta}(t)dt \\
\sigma_{\epsilon_{R_\theta}} &= \frac{1}{T} \int_0^T (\epsilon_{R_\theta}(t) - \bar{\epsilon}_{R_\theta})dt.
\end{align*}$$

(1)

In Fig. 3 one can see how the tracking error obviously gets worse with the increasing of the noise intensity. However, for reasonable level of noise, the mean tracking error is always limited and sufficiently small.

Furthermore, and more importantly, the platform never gets unstable even if the measurements are extremely degraded.

D. Motor Time Constant Robustness

To test the robustness considering non-ideal motors, we modeled each of them as a first order system characterized by a time constant $\tau_M \in \mathbb{R}_{>0}$. In Fig. 4 we assess the robustness with respect to it. In particular we plot $\bar{\epsilon}_{pR}$, $\sigma_{\epsilon_{pR}}$, $\bar{\epsilon}_{R_\theta}$ and $\sigma_{\epsilon_{R_\theta}}$, varying the time constant from the value of 0.01 $[s]$ to the one of 0.1 $[s]$. As expected, the tracking performance gets worse until the system becomes unstable for time constants larger than 0.1 $[s]$. For larger values one could easily incorporate the motor dynamics in the model, including the motor speed in the system state and considering its derivative as the new input. Being the new model fully controllable, a design similar to the one presented in [1] would make the job of stabilizing the platform. However, for a standard brushless motors with the closed-loop speed controller presented in [2], the time constant is about 0.03 $[s]$. For this value the corresponding tracking error is sufficiently good without the need of an extended model.

E. Motor Communication Delay Robustness

In a real platform there will always be a certain delay in the communication with the motor controller. We have tested which is the maximum delay value for which we can obtain a stable behavior. In Fig. 5 we show the tracking performance with respect to an increasing value of the delay between the commanded angular velocity for the motor and the one received as set-point by the motor controller. The tracking error, although sufficiently small, increases until the maximum delay of 0.07 $[s]$; after this value some oscillatory modes appear. However notice that a delay of 0.07 $[s]$ is incredibly large with respect to standard control input delays on aerial platform where the controller is implemented on an on-board PC. Usually, for those configurations, the delay value is below 0.002 $[s]$.
the vehicle to follow the same desired trajectory considered.

- Reduce the risk of saturation, we computed an omniplus design beyond the scope of this manuscript.
- Further improved using a disturbance observer. However, this performance under an external constant force acting at the position of the 7th propeller from time 2 [s] to time 10 [s].

**F. Disturbance Rejection**

We investigate here the behavior of the system under external disturbances. In particular, in Fig. 6 we show the tracking performance under an external constant force acting at the position of the 7th propeller from time 2 [s] to time 10 [s], generating both translation and orientation disturbances. This external force defined in world frame is equal to $[2 \ 2 \ 2]^T$ [N].

As we can see from Fig. 6 at time 2 [s] the external force is “activated” and the tracking error increases. However, thanks to the integral action in the controller, after a transient the system is able to counterbalance the effects of the disturbance bringing to zero the tracking error. A similar behavior is shown when the external force is “de-activated” at time 10 [s].

Notice that the disturbance rejection performance could be further improved using a disturbance observer. However, this goes beyond the scope of this manuscript.

**G. Non-optimized Omniplus Design**

To better show the importance of optimizing a design to reduce the risk of saturation, we computed an omniplus design without minimizing the condition number. The resulting platform is shown in Fig. 7 on the top. We then required the vehicle to follow the same desired trajectory considered for the optimized design. In Fig. 7 one can clearly see that the trajectory is tracked as for the optimized platform but the required control inputs are much larger and they go beyond the maximum value of 130 [Hz]. The simulation does not consider the saturation in order not to obtain unstable behaviors and shows the required control inputs for the full trajectory.

**H. Optimal Omniplus Design with 8 Propellers**

Finally to show that the proposed algorithm works with any number of propellers, we computed an optimal omniplus platform with $n = 8$. Figure 8 shows the obtained design from different perspectives and the tracking performance following the usual desired trajectory. One can see that the tracking and the desired control inputs are comparable to the one of the optimal omniplus design with 7 thrusters.

**References**


Fig. 8: Optimized omniplus design with $n = 8$ and $\text{cond}(F) = 3.59$, and relative control inputs required to track the desired trajectory.