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To cite this version:
Justin Carpentier. Analytical Inverse of the Joint Space Inertia Matrix. Rapport LAAS n° 18125. 2018. <hal-01790934>

HAL Id: hal-01790934
https://hal.laas.fr/hal-01790934
Submitted on 14 May 2018
Analytical Inverse of the Joint Space Inertia Matrix

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Abstract—In this short report, we briefly introduce a new algorithm to compute the analytical inverse of the joint space inertia matrix. Such inverse may be useful for several applications like the computation of the analytical derivatives of the forward dynamics. We also provide a free C++ implementation of this algorithm inside our C++ framework for rigid-body dynamics computations called Pinocchio.

I. CONTENT

The algorithm that we introduce now allows to compute in recursive manner (i.e. by exploiting the sparsity induced by the kinematic tree) the inverse of the joint space inertia matrix, without explicitly computing the joint space inertia matrix itself. It formally exploits the computations done in the articulated body algorithms by removing the affine terms. To the best of our knowledge, this is the first time that this algorithm is introduced and detailed.

This algorithm is composed of three passed, similarly to ABA. We use the same notations that have been used so far in our paper on the analytical derivatives [1] and which follows the ones introduced in [4]. We denote by \( M^{-1} \) the inverse of \( M \) et we use the notation \( M_{\text{inv}}[i,:] \) to designate the rows of \( M_{\text{inv}} \) which correspond to the joint \( i \). Using this notation, \( M_{\text{inv}}[i, \text{subtree}(i)] \) are the columns of \( M_{\text{inv}}[i,:] \) supported the joint \( i \) including \( i \) itself. We also exploit the fact that \( M^{-1} \) is symmetric.

In Algo. 1, \( F_i \) is the force-set collecting the contributions of the supporting tree rooted at \( i \) and \( P_i \) is a motion-set which contains the contributions of all the parents of joint \( i \).

II. IMPLEMENTATION

This algorithm is made freely available in our C++ framework called Pinocchio [3]. Pinocchio allows to compute in efficient manner the forward and inverse dynamics of poly-articulated systems such as humanoid robots, robotic manipulators, quadrupedal robots. Pinocchio is now at the hearth of many softwares developed in the Gepetto team at LAAS-CNRS for planing and control of robots [5] [2]. In addition, Pinocchio comes with a complete Python interface for easy prototyping.

ACKNOWLEDGMENTS

This work is supported by the RoboCom++ FLAG-ERA JTC 2016 proposal.

REFERENCES

Algorithm 1 - Pseudo code of the algorithm to directly compute the inverse of the joint space inertia matrix and which is inspired from ABA exposed by Featherstone [4, p. 132] and follows the same notations.

1 First forward pass:
for \( i = 1 \) to \( N_B \) do
2 \([X_J, S_i] = \text{jcalc(jtype}(i), q_i, \dot{q}_i)\)
3 \(^iX_{\lambda(i)} = X_J X_T(i)\)
4 \( I_{i}^A = I_i \)
end

7 Backward pass:
8 for \( i = N_B \) to \( 1 \) do
9 \( U_i = I_i^A S_i \)
10 \( D_i = S_i^T U_i \)
11 if \( i \) is not a leaf then
12 \( M_{\text{inv}}[i, \text{subtree}(i)] = \mathbf{D}_i^{-1} S_i^T F\lambda(i)[; \text{subtree}(i)] \)
13 if \( \lambda(i) \neq 0 \) then
14 \( F\lambda(i)[; \text{subtree}(i)] = U_i M_{\text{inv}}[i, \text{subtree}(i)] \)
15 \( F_{\lambda(i)}[][; \text{subtree}(i)] = F_{\lambda(i)}[; \text{subtree}(i)] + \lambda(i) X_i^T F\lambda(i)[; \text{subtree}(i)] \)
end
17 else
18 \( F\lambda(i)[; i] = F_{\lambda(i)}[; i] + \lambda(i) X_i^T U_i M_{\text{inv}}[i, i] \)
end
21 \( M_{\text{inv}}[i, i] = \mathbf{D}_i^{-1} \)
22 if \( \lambda(i) \neq 0 \) then
23 \( I_{i}^A = I_i^A - U_i D_i^{-1} U_i^T \)
24 \( I_{\lambda(i)}^A = I_{\lambda(i)}^A + \lambda(i) X_i^T I_i^{-1} X_{\lambda(i)} \)
end
26 Second forward pass:
28 for \( i = 1 \) to \( N_B \) do
29 if \( \lambda(i) \neq 0 \) then
30 \( M_{\text{inv}}[i, \text{subtree}(i)] = M_{\text{inv}}[i, \text{subtree}(i)] - \mathbf{D}_i^{-1} U_i^T X_{\lambda(i)} P_{\lambda(i)}[; \text{subtree}(i)] \)
end
32 \( P_i = S_i M_{\text{inv}}[i, \text{subtree}(i)] \)
33 if \( \lambda(i) \neq 0 \) then
34 \( P\lambda(i)[; \text{subtree}(i)] = P_i[; \text{subtree}(i)] + \lambda(i) X_i P_{\lambda(i)}[; \text{subtree}(i)] \)
end