A New Mixed Time Framework for the Periodically Aggregated Resource-Constrained Project Scheduling Problem

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1 Introduction

We consider a project scheduling problem in a context such that temporal aspects (start and completion of activities, precedence between activities) are handled precisely, while resource consumption is evaluated more roughly over aggregated periods. Hence, the Periodically Aggregated Resource-Constrained Project Scheduling Problem (PARCPSP) is well-suited for an intermediate planning/scheduling decision level, in particular with human resources or energetic resources. This problem, introduced in [1], has been modelled by means of a Mixed Integer Linear Programming (MILP) formulation, based on a mixed time framework. In this abstract, we present an alternative way to manage both continuous and discrete temporal representations simultaneously, in order to derive a new MILP formulation for a purpose of theoretical and computational comparison.

2 Periodically Aggregated Resource-Constrained Project Scheduling Problem

The problem consists in finding a non-preemptive schedule that minimises the execution duration of a project (1) under precedence constraints (2) and periodically aggregated resource constraints (3). The project is defined by a set $\mathcal{A}$ of n activities and a set $\mathcal{R}$ of m renewable resources. At each instant of its execution, activity i (processing time p_i) requires a fix amount ($r_{i,k}$) on resource k (capacity b_k). Precedence relations are represented by an activity pair list (E). The time horizon is subdivided into periods of parameterized length Δ (positive real). An abstract formulation for the PARCPSP is:

$$\text{Minimise} \quad S_{n+1} - S_0 \tag{1}$$

$$\text{s.t.} \quad S_{i_2} - S_{i_1} \geq p_{i_1} \qquad \forall (i_1, i_2) \in E \tag{2}$$

$$\sum_{i \in \mathcal{A}} r_{i,k} \frac{d_{i,\ell}(S)}{\Delta} \leq b_k \qquad \forall k \in \mathcal{R} \quad \forall \ell \in \mathbb{Z} \tag{3}$$

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Where: S_i is the start date of activity i, S_0 and S_{n+1} denote respectively the least start date and the greatest completion date, and $d_{i,\ell}(S)$ is the execution duration of activity i in period ℓ, i.e. the length of the intersection of two time intervals: the ℓ^{th} period (i.e. $[(\ell-1)\Delta, \ell\Delta]$) and the execution interval of activity i (i.e. $[S_i, S_i + p_i]$). Hence, the term $r_{i,k}\frac{d_{i,\ell}(S)}{\Delta}$ is the average demand of activity i on resource k in period ℓ.

$$d_{i,\ell}(S) = \max(0, \min(S_i + p_i, \ell\Delta) - \max(S_i, (\ell-1)\Delta))$$

3 A new mixed time framework

Two temporal representations coexist. On the one hand, start dates S_i permit to handle events and precedence exactly (continuous time representation). On the other hand, lengths $d_{i,\ell}(S)$ have to be computed in every period ℓ to evaluate the resource usage (discrete time representation). In terms of modelling, how to ensure a cohesion between these two temporal representations?

The initial MILP introduced in [1], adapted from [2], uses binary variables with an increasing step behaviour, and translate 6 out of 13 relations from Allen's algebra [3] (abstracts all possible relative positioning of two time intervals) into linear constraints. In this first model, constraints with a big-M term are required.

We design another approach to compute $d_{i,\ell}(S)$ values. This second model involves more (continuous) variables but less constraints, none of which have a big-M term.

Suppose the planning horizon contains L consecutive periods $\{1, \cdots, L\}$, so that the time interval covered is $[0, L\Delta]$. For every activity i, in every period ℓ (time interval $[(\ell-1)\Delta]$), two lengths are considered: the length $\lambda_{i,\ell}(S)$ of the intersection of the period and $(0, S_i]$, and the length $\mu_{i,\ell}(S)$ of the intersection of the period and $[S_i + p_i, L\Delta)$.

$$\lambda_{i,\ell}(S) = \max(0, \min(\Delta, S_i - (\ell-1)\Delta)) \qquad \mu_{i,\ell}(S) = \max(0, \min(\Delta, \ell\Delta - S_i - p_i))$$

The new mixed time framework relies on the fact that the three intervals whose length is measured by $\lambda_{i,\ell}(S)$, $d_{i,\ell}(S)$ and $\mu_{i,\ell}(S)$ form a partition of period ℓ (see Fig. 1).

$$\lambda_{i,\ell}(S) + d_{i,\ell}(S) + \mu_{i,\ell}(S) = \Delta$$

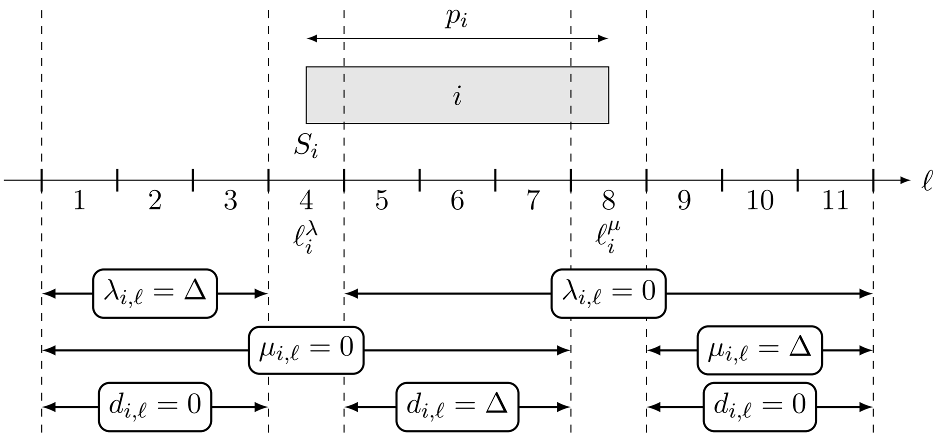

Figure 1: Principle of the new mixed time framework

Let $\ell_i^\lambda(S)$ (respectively $\ell_i^\mu(S)$) the index of the period that contains S_i (respectively $S_i + p_i$). Note that the values of $\lambda_{i,\ell}(S)$ and $\mu_{i,\ell}(S)$ are equal to 0 or Δ in almost every period ℓ, with a decreasing (respectively increasing) step behaviour.

$$\lambda_{i,\ell}(S) \begin{cases} = \Delta & \text{if } \ell < \ell_i^\lambda(S) \\ \in [0, \Delta] & \text{if } \ell = \ell_i^\lambda(S) \\ = 0 & \text{if } \ell > \ell_i^\lambda(S) \end{cases} \qquad \mu_{i,\ell}(S) \begin{cases} = 0 & \text{if } \ell < \ell_i^\mu(S) \\ \in [0, \Delta] & \text{if } \ell = \ell_i^\mu(S) \\ = \Delta & \text{if } \ell > \ell_i^\mu(S) \end{cases}$$

The values of $\ell_i^\lambda(S)$ and $\ell_i^\mu(S)$ are encoded by binary variables $z_{i,\ell}^\lambda(S)$ and $z_{i,\ell}^\mu(S)$ that follow the same step behaviour.

$$z_{i,\ell}^\lambda(S) \begin{cases} = 1 & \text{if } \ell \leq \ell_i^\lambda(S) \\ = 0 & \text{otherwise} \end{cases} \qquad z_{i,\ell}^\mu(S) \begin{cases} = 1 & \text{if } \ell \geq \ell_i^\mu(S) \\ = 0 & \text{otherwise} \end{cases}$$

Thus, linear lower and upper bounds on $\lambda_{i,\ell}(S)$ and $\mu_{i,\ell}(S)$ can be derived (note that a step behaviour of the binary variables is enforced by transitivity).

$$z_{i,\ell+1}^\lambda(S) \leq \frac{\lambda_{i,\ell}(S)}{\Delta} \leq z_{i,\ell}^\lambda(S) \qquad z_{i,\ell-1}^\mu(S) \leq \frac{\mu_{i,\ell}(S)}{\Delta} \leq z_{i,\ell}^\mu(S)$$

It remains to ensure that each activity i is processed during exactly p_i time units; by doing so, the values of $\lambda_{i,\ell_i^\lambda(S)}(S)$ and $\mu_{i,\ell_i^\mu(S)}(S)$ are implicitly balanced.

$$\sum_{\ell=1}^{L} d_{i,\ell}(S) = \sum_{\ell=1}^{L} (\Delta - \lambda_{i,\ell}(S) - \mu_{i,\ell}(S)) = p_i$$

Finally, S_i can indeed be inferred directly from either $\lambda_{i,\ell}(S)$ or $\mu_{i,\ell}(S)$.

$$S_i = \sum_{\ell=1}^{L} \lambda_{i,\ell}(S) \quad \Leftrightarrow \quad S_i + p_i = \sum_{\ell=1}^{L} \lambda_{i,\ell}(S) + \sum_{\ell=1}^{L} d_{i,\ell}(S) = L\Delta - \sum_{\ell=1}^{L} \mu_{i,\ell}(S)$$

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