Automotive vehicle sideslip angles estimation in a bounded-error context

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Abstract:
This paper concerns the estimation of the sideslip dynamics of automotive vehicles in a bounded-error framework. This work considers the linear varying parameter (LPV) model of an automotive vehicle in a polytopic form. This model is obtained by considering the dependency of the model with respect to the vehicle speed as a varying parameter.
Since this parameter belongs to a bounded interval (the vehicle speed is obviously bounded by constructor constraints), interval analysis tools propose efficient approaches for estimating the vehicle unmeasured dynamics in all situations. Indeed, in this paper, an estimation strategy is developed to reconstruct the vehicle sideslip dynamical behaviour and then proceed to a stability analysis of the car based on the driving situation.
Simulations performed on a nonlinear vehicle model and its parameters (validated by an experimental procedure on a real Renault Megane Coupé) are used to validate the estimation strategy and highlight its efficiency.

Keywords: Estimation, Interval analysis, linear varying parameter (LPV) modeling, automotive systems, stability evaluation.

1. INTRODUCTION

In the last century, the automotive vehicle's industry has greatly evolved. Thanks to the significant technological developments, new generations of safe, efficient and ecological cars have been produced. This has led to an increasing competition between several car companies by continuously enhancing the performance of their products. For this sake, both academic and industrial communities have focused on proposing new strategies that enhance the vehicle behaviour. One of the most challenging issues to achieve this goal is the lack of informations about some of the vehicle's dynamics that can not be measured using conventional sensors. These informations may be crucial to evaluate the car's state (stability, driving situation,...) and to provide the adequate solutions (inputs to the control and diagnosis strategies).
Recently, a lot of works have been developed in this scope. In Gustafsson (1997), a strategy to estimate the vehicle slip based on tire friction is presented. It allows to alarm the driver for sudden changes. Another estimation strategy based on Kalman filtering has presented a simple implementable estimation for these dynamics in Venhovens and Naab (1999). In Ono et al. (2003), an on-line least-squares method was applied to wheel rotational velocities to estimate the parameters of the tire frictions and then the car's slip. An identification approach based on road type recognition is proposed in Guan et al. (2014). Other strategies have already tried to provide solutions to the considered issues (see Morrison and Cebon (2016); Du et al. (2014); Gadola et al. (2014)).
All the previously cited strategies present some limitations concerning the estimation of these non linear slip dynamics. The correlation between the vertical, lateral and longitudinal dynamics are very strong and make it very difficult to isolate the variable to be estimated.
For this purpose, authors have considered using interval state observers on a linear varying parameter model that approximates efficiently the considered non linear phenomenon. Indeed, an important framework which has been largely investigated to solve the problems of estimation for generic nonlinear systems is based on LPV transformations (for example Lee (1997); Shamma and Xiong (1999)). In Wang et al. (2012), interval observers are used to parameter estimate of nonlinear systems. There exist several approaches to equivalently represent a nonlinear system in a LPV form (for example Hecker and Varga (2004) or Marcos and Balas (2004)). It is worth to note that such a procedure is not based on approximate linearization. It is global and it transforms the nonlinear system by introducing extended parametric uncertainties to the LPV setting. There are several methods for estimating LPV systems, one of them is based on design of interval state observers Rassi et al. (2012) which provide two variables evaluating the lower and upper bounds for state values of LPV systems.
In this paper, a state observer based on interval analysis is used to estimate the sideslip dynamics of the automotive vehicle. This estimation allows us to establish an adequate analysis of the car's stability while performing several drinving scenarios. The proposed observers guarantee to enclose the set of system states that is consistent with the model, the disturbances and the measurement noise. Moreover, it is only assumed that the measurement noise and the disturbances are bounded without any additional information such as stationarity, uncorrelation or type of

distribution. The proposed strategy is applied the LPV model based on a non linear vehicle model validated by an experimental test procedure on a real Renault Mégane Coupé.
This paper is organised as follows: section 2 presents the non linear model of the automotive vehicle and the corresponding linear varying parameter model (LPV). In section 3, the estimation strategy based on the interval analysis state observer is presented. The next section presents simulation results obtained on the full non linear vehicle model validated on the real car. Finally, a conclusion on the proposed strategy and validation discusses the efficiency of the approach.

2. SYSTEM MODELING

In this section, a nonlinear model and its parameters validated on a real vehicle (Renault Mégane Coupé) for simulation and validation purposes is presented. Also, a linearized model and a linear varying parameter (LPV) model is proposed to cope with the estimation strategy requirements.

2.1 The model parameters

In the following, the vehicle model parameters obtained by an identification process on the real Renault Mégane Coupé are presented. Throughout the paper, indexes $i = \{f, r\}$ and $j = \{l, r\}$ are used to identify vehicle front, rear and left, right positions, respectively.

Symbol	Value	Unit	Signification
m	1535	kg	vehicle mass
I_z	2149	$kg.m^2$	vehicle yaw inertia
C_f	40000	N/rd	lateral tire front stiffness
C_r	40000	N/rd	lateral tire rear stiffness
S_r	12720	N	longitudinal tire rear stiffness
l_f	1.4	m	distance COG - front axle
l_r	1	m	distance COG - rear axle
t_r	1.4	m	rear axle length
R	0.3	m	tire radius
μ	$[2/5; 1]$	$-$	tire/road contact friction coefficient
v	$[50; 130]$	km/h	vehicle velocity coefficient

Table 1. Renault Mégane Coupé parameters.

2.2 The nonlinear model

A nonlinear full vehicle model has been validated by an experimental procedure on a real car . In this paper, only the nonlinear equations under interest that reproduce the vehicle lateral behaviour are presented (the full vehicle model with all the non linear equations describing its dynamical behaviour can be found in Fergani (2014)). The main equations that govern the lateral dynamics are the following:

$$\begin{cases} \dot{\beta} = (F_{ty_f} + F_{ty_r})/(mv) + \dot{\psi}, \\ \ddot{\psi} = \big[l_f(-F_{tx_f}\sin(\delta) + F_{ty_f}\cos(\delta)) - l_r F_{ty_r}, \\ \qquad -\Delta F_{tx_r} t_r + M_{dz}\big]/I_z, \end{cases} \tag{1}$$

where β is the sideslip angle and ψ is the vehicle yaw, F_{ty_f} represents lateral front tire forces, F_{ty_r} represents lateral rear tire forces and F_{tx_f} represent the longitudinal front tire forces, v is the vehicle speed, ΔF_{tx_r} is the differential rear braking force (obtained based on the braking torques $T_{b_{rj}}$), δ is the steering angle and M_{dz} is the yaw moment disturbance.

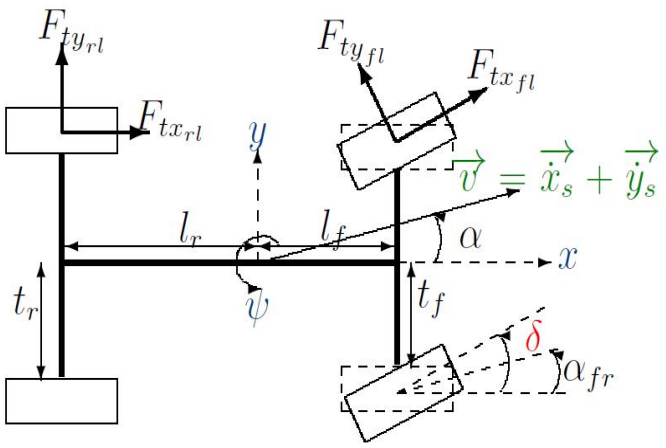

Fig. 1. View of the bicycle model.

Remark 2.1. It is worth noting that the sideslip dynamics are highly nonlinear and cannot be measured via a conventional sensor.

2.3 The linear bicycle model

Since the previously introduced model is highly non linear, a linear bicycle model as illustrated by Fig. 1 reproducing the lateral behaviour of the car is used for this study by linearizing (1). The model is obtain considering the following:

- Low sideslip angles: $|\beta| < 7$ degrees,
- Low longitudinal slip ratio: < 0.1,
- Low steering angles: $\cos(\delta) \simeq 1$.

The linearized lateral tire forces are:

$$\begin{cases} F_{ty_f} = C_f \beta_f, \\ F_{ty_r} = C_r \beta_r, \end{cases} \tag{2}$$

with β_f and β_r denote the front and rear sideslip angles,

$$\begin{cases} \beta_f = \delta - \beta - \dfrac{l_f \dot{\psi}}{v}, \\ \beta_r = \beta + \dfrac{l_f \dot{\psi}}{v}. \end{cases} \tag{3}$$

This leads to the following state space representation (4):

$$\begin{bmatrix}\dot{\beta}\\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix}\dfrac{-C_f - C_r}{mv} & 1+\mu\dfrac{-l_rC_r - l_fC_f}{mv^2}\\ \dfrac{-l_rC_r - l_fC_f}{I_z} & \dfrac{-l_f^2C_f - l_r^2C_r}{I_z v}\end{bmatrix}\begin{bmatrix}\beta\\ \dot{\psi}\end{bmatrix} + \begin{bmatrix}\dfrac{C_f}{mv} & 0 & 0 & 0\\ \dfrac{l_fC_f}{I_z} & \dfrac{1}{I_z} & \dfrac{S_rRt_r}{2I_z} & -\dfrac{S_rRt_r}{2I_z}\end{bmatrix}\begin{bmatrix}\delta\\ M_{dz}\\ T_{b_{rl}}\\ T_{b_{rr}}\end{bmatrix} \tag{4}$$

Remark 2.2. $\mu \in [0;1]$ is the tire/road adhesion coefficient. Its value depends on the road conditions (dry, wet, icy,...) and highly influences the lateral dynamics of the vehicle. The influence of this parameter is shown in Fig. 2 (more details in Poussot-Vassal (2008)):

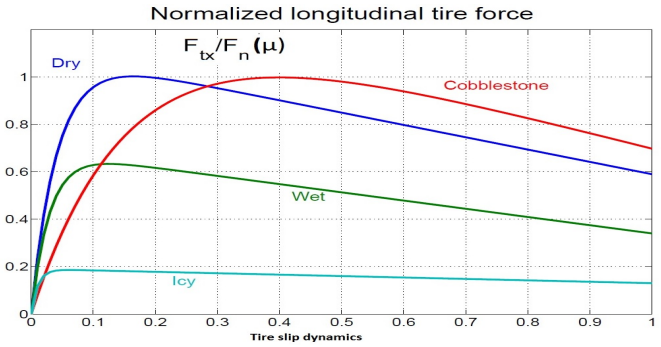

Fig. 2. Road adhesion influence on the tire and slip dynamics .

2.4 The linear varying parameter model (LPV)

To cope with the estimation strategy and to have better results, the model may have to be put in a linear varying parameter form (LPV model). Indeed, the choice of the varying parameter can be crucial for the strategy development. A close examination of the model (4) shows a dependency of the model w.r.t the vehicle speed v. This can be noticed on the two graphs of Fig. 3.

In this paper, the varying parameter is not chosen to be directly the speed v but $\frac{1}{v}$ such that $\rho_1 = \frac{1}{v}$ and $\rho_2 = \frac{1}{v^2}$. The result is then a polytopic LPV model as follows:

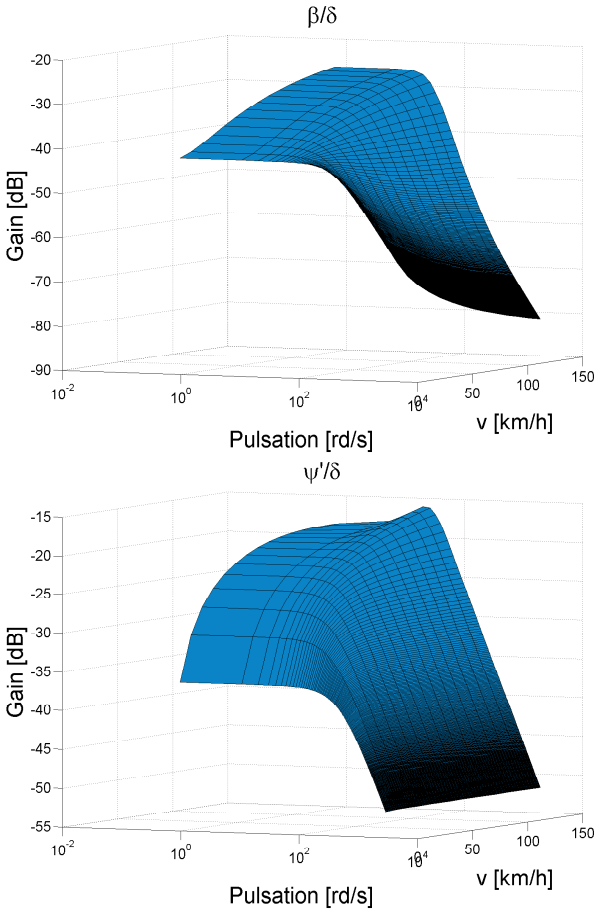

Fig. 3. Model dependency w.r.t vehicle speed.

$$\begin{bmatrix}\dot{\beta}\\ \ddot{\psi}\end{bmatrix} = \left(A_0 + \rho_1 A_1 + \rho_2 A_2\right)\begin{bmatrix}\beta\\ \dot{\psi}\end{bmatrix} + \left(B_0 + \rho_1 B_1\right)\begin{bmatrix}\delta\\ M_{dz}\\ T_{b_{rl}}\\ T_{b_{rr}}\end{bmatrix}$$

where,

$$A_0 = \begin{bmatrix}0 & 1\\ \mu\dfrac{-l_rC_r - l_fC_f}{I_z} & 0\end{bmatrix},$$

$$A_1 = \begin{bmatrix}\mu\dfrac{-C_f - C_r}{m} & 0\\ 0 & \mu\dfrac{-l_f^2C_f - l_r^2C_r}{I_z}\end{bmatrix},$$

$$A_2 = \begin{bmatrix}0 & \mu\dfrac{-l_rC_r - l_fC_f}{m}\\ 0 & 0\end{bmatrix},$$

$$B_0 = \begin{bmatrix}0 & 0 & 0 & 0\\ \dfrac{l_fC_f}{I_z} & \dfrac{1}{I_z} & \dfrac{\mu S_rRt_r}{2I_z} & -\dfrac{\mu S_rRt_r}{2I_z}\end{bmatrix},$$

$$B_1 = \begin{bmatrix}\dfrac{C_f}{m} & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}. \tag{5}$$

Remark 2.3. Since the selected varying parameter is the inverse of the vehicle speed v which does not depend on the state of the system, the resulting model is an LPV model and not a qLPV (quasi LPV is not as relevant as a total LPV model for a polytopic representation). Also, the vehicle speed is obviously bounded which is sweetable for this representation.

3. INTERVAL ANALYSIS ESTIMATION STRATEGY

3.1 Basic tools of interval analysis

A real interval $[u] = [\underline{u}, \overline{u}]$ is a closed and connected subset of R where $\underline{u}$ represents the lower bound of $[u]$ and $\overline{u}$ represents the upper bound.
The width of an interval $[u]$ is defined by:

$$w(u) = \overline{u} - \underline{u} \tag{6}$$

and its midpoint by:

$$m(u) = (\overline{u} + \underline{u})/2 \tag{7}$$

The set of all real intervals of R is denoted IR.

Two intervals $[u]$ and $[v]$ are equal if and only if $\underline{u} = \underline{v}$ and $\overline{u} = \overline{v}$.
Real arithmetic operations are extended to intervals Moore (1966). Arithmetic operations on two intervals $[u]$ and $[v]$ can be defined by:

$$\circ \in \{+, -, *, /\}, \;\; [u] \circ [v] = \{x \circ y \mid x \in [u], \; y \in [v]\}. \tag{8}$$

An interval vector (or box) $[X]$ is a vector with interval components and may equivently be seen as a cartesian product of scalar intervals:

$$[X] = [x_1] \times [x_2] ... \times [x_n]. \tag{9}$$

The set of $n-$dimensional real interval vectors is denoted by IR^n.
An interval matrix is a matrix with interval components. The set of $n \times m$ real interval matrices is denoted by $\mathrm{IR}^{n \times m}$. The width $w(.)$ of an interval vector (or of an interval matrix) is the maximum of the widths of its interval components. The midpoint $m(.)$ of an interval vector (resp. an interval matrix) is a vector (resp. a matrix) composed of the midpoint of its interval components.
Classical operations for interval vectors (resp. interval matrices) are direct extensions of the same operations for punctual vectors (resp. punctual matrices) Moore (1966).

Let $f : \mathrm{R}^n \to \mathrm{R}^m$, the range of the function f over an interval vector $[u]$ is given by:

$$f([u]) = \{f(x) | x \in [u]\}. \tag{10}$$

The interval function $[f]$ from IR^n to IR^m is an inclusion function for f if:

$$\forall [u] \in \mathrm{IR}^n, \quad f([u]) \subseteq [f]([u]). \tag{11}$$

An inclusion function of f can be obtained by replacing each occurrence of a real variable by its corresponding interval and by replacing each standard function by its interval evaluation.
Such a function is called the natural inclusion function. In practice the inclusion function is not unique, it depends on the syntax of f.

3.2 Bounded state Estimation

The following results were developed in Efimov et al. (2013). Given an LPV system described by:

$$\dot{x} = [A_0 + \delta A(\rho(t))]x + b(t)u, \;\; y = Cx. \tag{12}$$

Define by $x^+ = \max(x, 0)$, $x^- = x^+ - x$ and $A^+ = \max(A, 0)$, $A^- = A^+ - A$. Thus only nonegative vectors and matrices are used.

The observer structure is described by:

$$\dot{\underline{x}} = [A_0 - LC]\underline{x} + [\underline{\Delta A}^+ \, \underline{x}^+ - \overline{\Delta A}^+ \, \underline{x}^- - \underline{\Delta} A^- \overline{x}^+ + \overline{\Delta A}^- \, \overline{x}^-] + Ly + \underline{b}(t), \tag{13}$$

and:

$$\dot{\overline{x}} = [A_0 - LC]\overline{x} + [\overline{\Delta A}^+ \, \overline{x}^+ - \underline{\Delta A}^+ \, \overline{x}^- - \overline{\Delta} A^- \underline{x}^+ + \underline{\Delta} A^- \underline{x}^-] + Ly + \overline{b}(t), \tag{14}$$

Theorem 1. Assume that the state x is bounded and that $(A_0 - LC) \in \mathcal{M}$. Then, the observer structure proposed above is an interval observer for the LPV system if $\underline{x}(0) \leq x(0) \leq \overline{x}(0)$. In addition, if there exist a matrix $P \in \mathbb{R}^{n \times n}$, $P = P^T \succ 0$ and $\gamma > 0$ such that the following Riccati equation is satisfied:

$$G^T P + PG + 2\gamma^{-2} P^2 + 4\gamma^2 \mu^2 I_{2n} + Z^T Z \prec 0 \tag{15}$$

then $\underline{x}, \overline{x} \in \mathcal{L}_\infty^n$ where $\mu = ||\overline{\Delta A} - \underline{\Delta A}||_2$ and:

$$G = \begin{bmatrix} A_0 - LC + \underline{\Delta A}^+ & -\underline{\Delta A}^- \\ -\overline{\Delta} A^- & A_0 - LC + \overline{\Delta A}^+ \end{bmatrix} \tag{16}$$

4. SIMULATION RESULTS

The following simulation results are the ones obtained using a full vehicle nonlinear model validated by an experimental procedure on a real Renault Mégane Coupé.
The scenario used for the simulation is the as follows:

- The vehicle is running at 100km/h on a wet road (the tire road contact friction coefficient decreases).
- The driver performs a double line change manoeuvre.
- A disturbing lateral wind occurs at vehicle's front.

Using the estimation strategy previously defined in section 3, the following results are obtained.
Fig. 4 represents the estimation of the sideslip dynamics of the car. This estimation is very important to evaluate

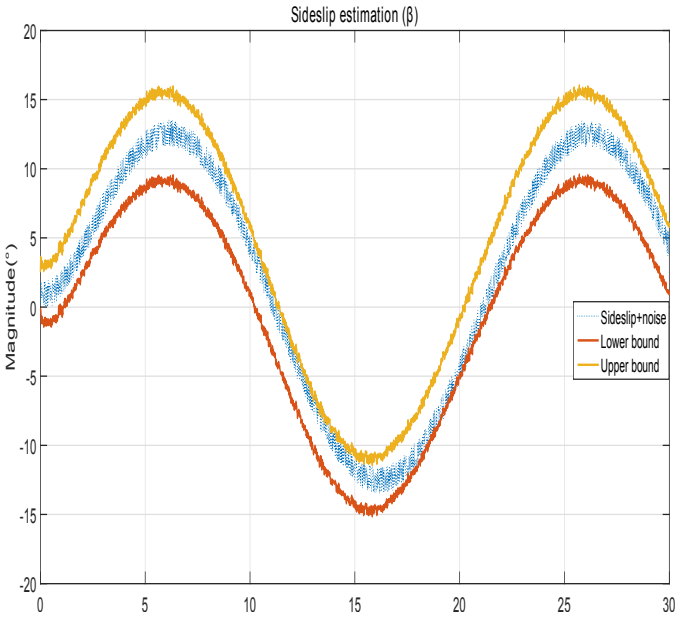

Fig. 4. Sideslip estimation using interval analysis.

the behaviour and the stability of the vehicle. This result shows the main result of the paper. The interval analysis estimation gives a good approximation of the sideslip angles and a guaranteed estimation envelope of these dynamics, which allows to evaluate the vehicle stability.

The vehicle stability is indeed directly correlated to

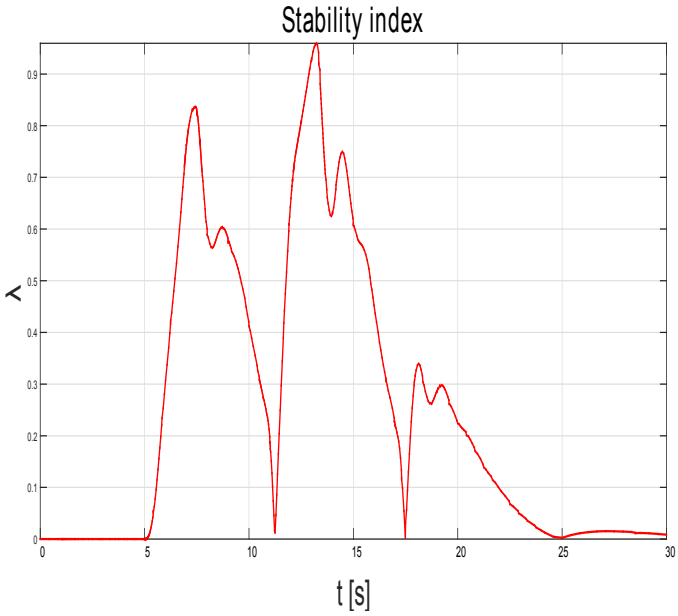

Fig. 5. Stability index λ during the manoeuver.

the vehicle sideslip dynamics (β). The stability index $\lambda = \left|2.49\dot{\beta} + 9.55\beta\right|$ gives information about the vehicle stability in the different driving situations. The result of Fig. 5 shows the vehicle stability considering the estimated sideslip angles. The vehicle stability region is derived from the phase-plane ($\beta - \dot{\beta}$) as follows:

$$\lambda < 1, \tag{17}$$

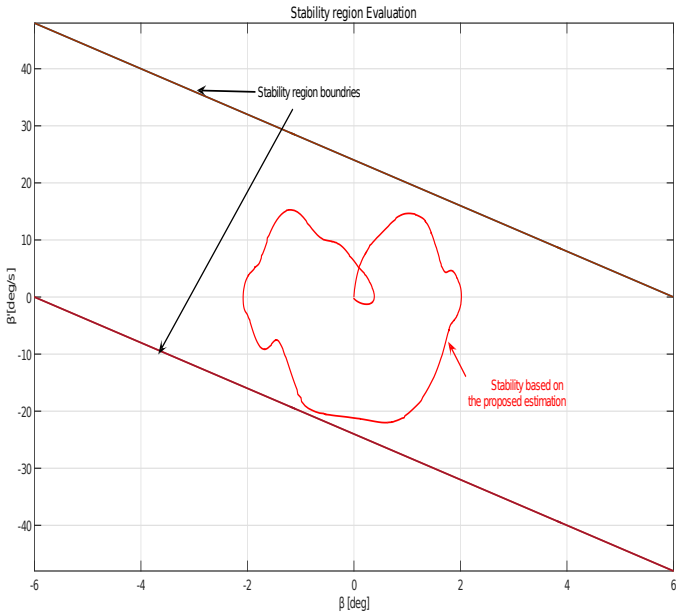

Fig. 6. Evolution within the stability region: β-$\dot{\beta}$ plane.

One of the motivations of the previously defined interval analysis estimation of the sideslip angle is shown in Fig. 6. It allows to analyse the evolution of the vehicle in the β-$\dot{\beta}$ plane. This result emphasizes the efficiency and importance of the proposed estimation strategy.

5. CONCLUSION

This paper has presented an automotive vehicle sideslip angles estimation in a bounded-error context. An estimation based on an interval observer has been developed using interval analysis tools and applied on a bicycle LPV model reproducing the main dynamical behaviour of the vehicle. It allows to estimate the lateral slip dynamics of the car. Indeed, the sideslip dynamics of the car, highly non linear, can not be metered using conventional sensors. These informations are crucial to vehicle stability analysis and evaluation.

Simulations performed on the full non linear model of Renault Mégane Coupé proves the efficiency of the proposed strategy.

Next step will be to optimize the experimental conditions (see Li2 (2016)) using optimal inputs and then provide a global control strategy based on this optimal estimation approach.

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