Flotation Process Fault Diagnosis Via Structural Analysis

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Abstract: For the improvement of safety and efficiency, fault diagnosis becomes increasingly important in mining industry. The expansion of flotation processes with high-tonnage cooper concentrators demands the use of large flotation circuits in which the large amount of instrumentation and interconnected subsystems (with coupled measured and non-measured variables) makes this process complex. Moreover, in a flotation process, any equipment failure can lead to a fault condition, which will affect the operation of this process. This paper proposes an approach for on-line fault diagnosis useful for a large flotation circuit based on a distributed architecture. In this approach, structural analysis is used for the design of the distributed fault diagnosis system. Finally, a procedure for the implementation of local diagnosers for on-line operation is presented and illustrated with an application to a flotation process.

Keywords: Fault diagnosis, Flotation process, Distributed architecture, Structural analysis

1. INTRODUCTION

Nowadays, the recovery is one of the most important process in mining industry. Currently, the recovery of minerals in this industry is mainly made through the flotation processing technique around the world.

Froth flotation uses the difference in surface properties to physically separate minerals from gangue and is one of the most widely used methods of ore concentration. In order to improve the recovery of valuable minerals, industrial flotation practice uses multiple cells. These cells are arranged in series forming a bank. A combination of banks is referred as flotation circuit. It is common for conventional flotation cells to be assembled in a circuit, with rougher, cleaner, and scavenger cells, which can be arranged in a designed configuration. On the other hand, in recent decades, the expansion of flotation with high-tonnage copper concentrators in Peru, Chile, etc. (O'Connell et al., 2016), has been demanding the use of large flotation circuits consisting of a large number of banks, with several cells each one.

Flotation equipment requires a machine for mixing and dispersing air throughout the mineral slurry while removing the froth product. Instrumentation is also necessary for a successful implementation of control strategies. The ultimate aim of control is to increase the economic efficiency of the process by seeking to optimise performance, and there are several strategies which can be adopted to achieve this, (Wills, 2006). In the flotation process, any equipment failure (in valves, sensors, pipelines, etc.), can lead to a fault condition, which will affect the operation of this process. In (Xu et al., 2012; Ming et al., 2015), methodologies for fault detection in flotation process operation that use analysis of variables measurement are proposed. The use of Principal Component Analysis (PCA) models is proposed in (Bergh and Acosta, 2009) to detect instrumentation failures on a flotation column. The development of fault diagnosis systems in mining industry is very important because an effective diagnosis of faults may have a high economic and safety impact. However, fault diagnosis in large flotation circuits is a difficult task due not only to the large amount of instrumentation, but also to its interconnected subsystems with coupled (measured and non-measured) variables between them. In this case, the implementation of a global diagnoser may be an impractical option because of the amount of needed communication, (Blanke et al., 2016). Thus the use of centralized architecture for on-line fault diagnosis can be very expensive and lack robustness for large-scale interconnected subsystems, (Pérez-Zuniga et al., 2018). One possibility to overcome this difficulty is to employ a distributed diagnosis architecture.

Recently, a distributed diagnosis framework for physical systems with continuous behavior using structural model has been proposed in (Bregon et al., 2014) and a distributed diagnosis approach with a set of diagnosers that are as local as possible was presented in (Khorasgani et al., 2015). In distributed diagnostic architectures, unlike centralized ones, it is not mandatory to know the model of the global system. Distributed architectures use subsystem models for diagnosis and local diagnosers (LDs), so they would be more appropriate for complex systems, (Pérez-Zuniga et al., 2017), such as the large flotation circuits.

The aim of this paper is to propose an approach for on-line fault diagnosis in flotation process circuit based on a distributed architecture propose in (Pérez-Zuniga et al., 2017). In this approach, structural analysis is used as an efficient tool for the design of fault diagnosis systems for nonlinear processes, (Isermann, 2006). Likewise, in order to optimize the offline design of LDs, Fault-Driven Minimal Structurally Overdetermined (FMSO) sets are calculated and guarantee minimal redundancy of analytical redundancy relations (ARR) generators, (Pérez-Zuniga et al., 2015). At last, a procedure for the residual generation for on-line operation is presented and shown with the flotation process.

2. PROBLEM STATEMENT

In a flotation process, the pulp is introduced into the first cell, the froth is collected through launders and the remaining pulp flows to the next cell. The magnitude of the flow depends on the pressure difference between two adjacent cells, the position of the control valves, and the viscosity and density of the pulp. Figure 1 shows the flotation process under study.

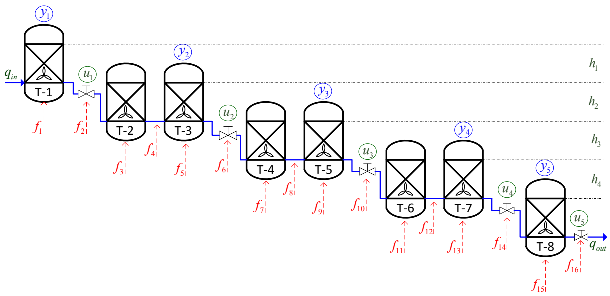

Fig. 1. Diagram of the flotation process under study.

Due to the physical characteristics of the flotation process, and considering the disturbances caused by the composition of the minerals and the constant and arduous work of the system, these systems usually have a limited efficiency, which is evidenced by faults in sensors, actuators and the system such as leaks in tanks and pipes, (Jamsa et al., 2003).

For the application of the structural analysis approach, let the system description consist of a set of n equations involving a set of variables partitioned into a set Z of n_Z known (or measured) variables and a set X of n_X unknown (or unmeasured) variables. We refer to the vector of known variables as z and the vector of unknown variables as x. The system may be impacted by the presence of n_f faults that appear as parameters in the equations. The set of faults is denoted by F and we refer to the vector of faults as $\mathtt{f}$.

Definition 1. (System). A system, denoted $\Sigma(z, x, \mathtt{f})$ or Σ for short, is any set of equations relating z, x and $\mathtt{f}$. The equations $e_i(z, x) \subseteq \Sigma(z, x, \mathtt{f})$, $i = 1, \ldots, n$, are assumed to be differential or algebraic in z and x.

The flotation process under study has 5 levels at different altitudes (h_1 to h_5) and is composed of 41 equations (36 for the system and 5 linked to the level control of each stage). Later, we assumed each level with outlet pipe as a subsystem so this system is composed by 5 subsystems. The flow q_{in} refers to the pulp inflow, while the flow q_{out} is related to the tailings. There are a set of 5 measurements y_1 to y_5 and a set of 5 control valves u_1 to u_5.

3. BACKGROUND THEORY

In this section, we summarize some important concepts presented in previous works related to the generation of diagnostic tests using structural analysis. Structural analysis allows to obtain structural models that are very useful for the design of Model Based Diagnosis (MBD) systems. The main assumption is that each system component is described by one or several constraints; thereby, violation of at least one constraint indicates that the system component is faulty.

The *structural model* of the system $\Sigma(z, x, \mathtt{f})$, also denoted with some abuse of notation by $\Sigma(z, x, \mathtt{f})$ or Σ in the following, can be obtained abstracting the functional equations. It retains a representation of which variables are involved in the equations. This abstraction leads to a bipartite graph $G(\Sigma \cup X \cup Z, A)$, or equivalently to $G(\Sigma \cup X, \mathcal{A})$, where $\mathcal{A} \subseteq A$ and $\mathcal{A}$ is a set of edges such that $a(i, j) \in \mathcal{A}$ iff variable x_i is involved in equation e_j.

The *structural model* $\Sigma(z, x, \mathtt{f})$ for this system is composed of 41 equations e_1 to e_{41} relating the known variables $Z = \{u_1, u_2, ..., u_5, y_1, y_2, ..., y_5, q_{in}, q_{out}\}$, the unknown variables $X = \{\dot{x}_1, x_1, \dot{x}_2, x_2, \dot{x}_3, x_3, ..., \dot{x}_8, x_8, q_0, q_1, q_2, ..., q_8\}$ and the set of sensors, actuators and process faults $F = \{f_1, f_2, f_3, f_4, f_5, ..., f_{16}\}$.

3.1 Analytical Redundancy Relations

Analytical redundancy relations (ARR) are equations that are deduced from an analytical model and only involve measured variables.

Definition 2. (ARR for $\Sigma(z, x, \mathtt{f})$). Let $\Sigma(z, x, \mathtt{f})$ be a system. Then, a relation $arr(z, \dot{z}, \ddot{z}, ...) = 0$ is an ARR for $\Sigma(z, x, \mathtt{f})$ if for each z consistent with $\Sigma(z, x, \mathtt{f})$ the relation is fulfilled.

Definition 3. (Residual generator for $\Sigma(z, x, \mathtt{f})$). A system taking a subset of the variables z as input, and generating a scalar signal arr as output, is a residual generator for the model $\Sigma(z, x, \mathtt{f})$ if, for all z consistent with $\Sigma(z, x, \mathtt{f})$, it holds that $\lim_{t \to \infty} arr(t) = 0$.

We use the decomposition of Dulmage Mendelshon as a tool to compute redundant sets using structural analysis, (Dulmage and Mendelsohn, 1958). Making use of this permutation, a system model Σ can be divided into three parts: the *structurally overdetermined* (SO) part Σ^+ with more equations than unknown variables; the *structurally just determined* part Σ^0, and the *structurally underdetermined* part Σ^- with more unknown variables than equations, (**?**).

Definition 4. (Structural redundancy). The structural redundancy $\rho_{\Sigma'}$ of a set of equations $\Sigma' \subseteq \Sigma$ is defined as the difference between the number of equations and the number of unknown variables in Σ'.

Definition 5. (Fault support). The fault support $F_{\Sigma'}$ of a set of equations $\Sigma' \subseteq \Sigma$ is defined as the set of faults that are involved in the equations of Σ'.

Definition 6. (PSO and MSO sets). A set of equations Σ is proper structurally overdetermined (PSO) if $\Sigma = \Sigma^+$ and minimally structurally overdetermined (MSO) if no proper subset of Σ is overdetermined (Krysander et al. (2010)).

Since PSO and MSO sets have more equations than variables, they can be used to generate ARRs and residuals.

A Fault-Driven Minimal Structurally Overdetermined (FMSO) set can be defined as an MSO set of $\Sigma(z, x, \mathtt{f})$ whose fault support is not empty.

Let us define $Z_\varphi \subseteq Z$, $X_\varphi \subseteq X$, and $F_\varphi \subseteq F$ as the set of known variables, unknown variables involved in the FMSO set φ, and its fault support, respectively. Next, we summarize the definition of FMSO set,

Definition 7. (FMSO set). A subset of equations $\varphi \subseteq \Sigma(z, x, \mathtt{f})$ is an FMSO set of $\Sigma(z, x, \mathtt{f})$ if (1) $F_\varphi \neq \emptyset$ and $\rho_\varphi = 1$ that means $|\varphi| = |X_\varphi| + 1$, (2) no proper subset of φ is overdeterminated. (Pérez-Zuniga et al., 2017)

We propose the use of FMSO sets that guarantee to always be impacted to faults contrary to the MSO sets that not may not be impacted by faults. Based on the concept of FMSO set, we summarize the concept of *detectable fault*, and *isolable fault*:

Definition 8. (Detectable fault). A fault $f \in F$ is detectable in the system $\Sigma(z, x, \mathtt{f})$ if there is an FMSO set $\varphi \in \Phi$ such that $f \in F_\varphi$.

Definition 9. (Isolable fault). Given two detectable faults f_j and f_k of F, $j \neq k$, f_j is isolable from f_k if there exists an FMSO set $\varphi \in \Phi$ such that $f_j \in F_\varphi$ and $f_k \notin F_\varphi$.

Additionally, a *Clear Minimal Structurally Overdetermined* (CMSO) set is a MSO set of $\Sigma(z, x, \mathtt{f})$ whose fault support is empty.

3.2 Distribution and Related Notions

A distributed diagnosis architecture assumes a decomposition of the process into subsystems, each with its corresponding LD, with similar functions and with possible communication between them. This communication must be properly designed; therefore, the local diagnoses are globally consistent. This architecture is shown in Figure 2.

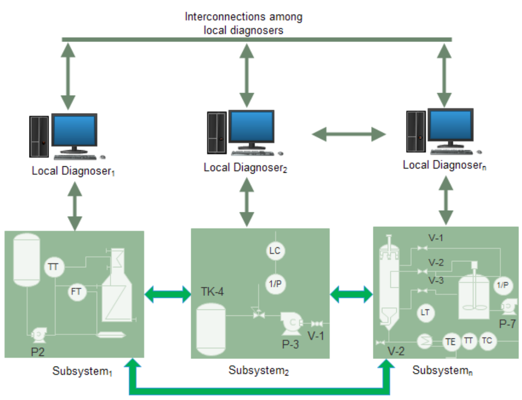

Fig. 2. Distributed diagnosis architecture

For the flotation process, in this paper we propose the design of the distributed system taking into account only the models of each subsystem to design LDs independently considering minimizing the communication between them until reaching the same diagnosis as with a centralized diagnosis. Let us consider the system Σ and define the following:

A decomposition of the system $\Sigma(z, x, \mathtt{f})$, into several subsystems $\Sigma_i(z_i, x_i, \mathtt{f}_i)$ is defined as a partition of its equations. Let $\Sigma(z, x, \mathtt{f}) = \{\Sigma_1(z_1, x_1, \mathtt{f}_1), ..., \Sigma_n(z_n, x_n, \mathtt{f}_n)\}$ with $\Sigma_i(z_i, x_i, \mathtt{f}_i) \subseteq \Sigma(z, x, \mathtt{f})$, $\bigcup_{i=1}^{n} \Sigma_i(z_i, x_i, \mathtt{f}_i) = \Sigma$, $\Sigma_i(z_i, x_i, \mathtt{f}_i) \neq \emptyset$ and $\Sigma_i(z_i, x_i, \mathtt{f}_i) \cap \Sigma_j(z_j, x_j, \mathtt{f}_j) = \emptyset$ if $i \neq j$. where z_i is the vector of known variables in Σ_i, x_i is the vector of unknown variables in Σ_i and $\mathtt{f}_i$ is the vector of faults in Σ_i. The set of variables and faults of the ith subsystem Σ_i, denoted as X_i, Z_i, and F_i respectively, are defined as the subset of variables of X, Z, and F respectively, that are involved in the subsystem $\Sigma_i(z_i, x_i, \mathtt{f}_i)$ also denoted by Σ_i.

For the flotation process, we consider each level as a subsystem, therefore, the first subsystem includes a tank and the outlet pipe, the second to the fourth subsystems, contain 2 tanks, the pipe between them and the outlet pipe and the fifth subsystem includes a tank and the outlet pipe, see Table 1.

Table 1. Model decomposition of the flotation process system into subsystems $\Sigma_i(z_i, x_i, \mathtt{f}_i)$, $i = 1, 2, 3, 4, 5$.

$\Sigma_1 = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$	$F_1 = \{f_1, f_2\}$
$X_1 = \{\dot{x_1}, x_1, q_0, w_1\}$	$Z_1 = \{u_1, y_1, q_{in}\}$
$\Sigma_2 = \{e_8, e_9, e_{10}, ..., e_{16}\}$	$F_2 = \{f_3, f_4, f_5, f_6\}$
$X_2 = \{\dot{x_2}, x_3, \dot{x_3}, q_2, w_2\}$	$Z_2 = \{u_2, y_2\}$
$\Sigma_3 = \{e_{17}, e_{18}, e_{19}, ..., e_{25}\}$	$F_3 = \{f_7, f_8, f_9, f_{10}\}$
$X_3 = \{\dot{x_4}, x_5, \dot{x_5}, q_4, w_3\}$	$Z_3 = \{u_3, y_3\}$
$\Sigma_4 = \{e_{26}, e_{27}, e_{28}, ..., e_{34}\}$	$F_4 = \{f_{11}, f_{12}, f_{13}, f_{14}\}$
$X_4 = \{\dot{x_6}, x_7, \dot{x_7}, q_6, w_4\}$	$Z_4 = \{u_4, y_4\}$
$\Sigma_5 = \{e_{35}, e_{36}, e_{37}, ..., e_{40}\}$	$F_5 = \{f_{15}, f_{16}\}$
$X_5 = \{\dot{x_8}, q_8, w_5\}$	$Z_5 = \{u_5, y_5\}$

The set of local variables of the ith subsystem, denoted by X_i^l, is defined as the subset of variables of X_i that are only involved in the subsystem Σ_i.

Definition 10. (Shared variables). The set of shared variables of the ith subsystem, denoted as X_i^s, is defined as:

$$X_i^s = \bigcup_{j=1, j\neq i}^{n} (X_i \cap X_j) = X_i \setminus X_i^l \qquad (1)$$

The set of shared variables of the whole system Σ is denoted by X^s.

Without loss of generality, we consider that all known variables of Z_i are local to the subsystem Σ_i, for $i = 1, \ldots, n$. If the same input was applied to several subsystems, it could be artificially replicated.

3.3 Distributed FMSO sets

Definition 11. (Local FMSO set). φ is a local FMSO set of $\Sigma_i(z_i, x_i, \mathtt{f}_i)$ if φ is an MFSO set of $\Sigma(z, x, \mathtt{f})$ and if

$\varphi \subseteq \Sigma_i$, $X_\varphi \subseteq X_i$ and $Z_\varphi \subseteq Z_i^l$. The set of local FMSO sets of Σ_i is denoted by Φ_i^l. The set of all local FMSO sets is denoted by $\Phi^l = \bigcup_{i=1}^{n} \Phi_i^l$.

Definition 12. (Shared FMSO set). φ is a shared FMSO set of subsystem $\Sigma_i(z_i, x_i, \mathtt{f}_i)$ if φ is an FMSO set of $\tilde{\Sigma}_i(\tilde{z}_i, \tilde{x}_i, \tilde{\mathtt{f}}_i)$, where $\tilde{z}_i$ is the vector of variables in $\tilde{Z}_i = Z_i \cup X_i^s$, $\tilde{x}_i$ is the vector of variables in $\tilde{X}_i = X_i^l$, and $\tilde{\mathtt{f}}_i = \mathtt{f}_i$). The set of shared FMSO sets for Σ_i is denoted by Φ_i^s. The set of all shared FMSO sets is denoted by $\Phi^s = \bigcup_{i=1}^{n} \Phi_i^s$.

From the above definition, a shared FMSO set φ for subsystem $\Sigma_i(z_i, x_i, \mathtt{f}_i)$ is such that $\varphi \subseteq \Sigma_i$, $X_\varphi \subseteq X_i^l$, $Z_\varphi \cap X_i^s \neq \emptyset$, and $Z_\varphi \subseteq (Z_i \cup X_i^s)$.

Definitions 11 and 12 can also be applied to CMSO sets to define *local CMSO sets* Λ_i^l and *shared CMSO sets* Λ_i^s. The set of all shared CMSO sets is denoted by Λ^s.

Definition 13. (Compound FMSO set). A global FMSO set φ that includes at least one shared FMSO set $\varphi' \in \Phi_i^s$ is called a compound FMSO set. The set of compound FMSO sets of Σ_i is denoted by Φ_i^c. The set of all compound FMSO sets is denoted by $\Phi^c = \bigcup_{i=1}^{n} \Phi_i^c$.

Definition 14. (Root FMSO set). If a compound FMSO set $\varphi \in \Phi^c$ includes a shared FMSO set $\varphi' \in \Phi^s$, then φ' is a root FMSO set of φ.

Definition 15. (Locally detectable fault). $f \in F_i$ is locally detectable in the subsystem $\Sigma_i(z_i, x_i, \mathtt{f}_i)$ if there is an FMSO set $\varphi \in \Phi_i^l$ such that $f \in F_\varphi$.

Definition 16. (Locally isolable fault). Given two locally detectable faults f_j and f_k of F_i, $j \neq k$, f_j is locally isolable from f_k if there exists an FMSO set $\varphi \in \Phi_i^l$ such that $f_j \in F_\varphi$ and $f_k \notin F_\varphi$.

Some properties required for the generation of compound FMSO sets starting from shared FMSO sets are detailed in Pérez-Zuniga et al. (2017)

4. DISTRIBUTED DIAGNOSIS

First, a set of distributed local diagnosers (LD) that together make the entire system completely diagnosable through compound FMSO sets is obtained, then residual generators that make it possible to detect and isolate all system faults are implemented. First, Algorithm 1 for generating local diagnostics off-line is applied and then Algorithm 2 is proposed for on-line residual generation.

4.1 Offline distributed generation of LDs

The LD design is done off-line in Algorithm 1. First, local FMSO sets are computed for every subsystem Σ_i. If there is any fault not locally detectable, then a set of compound FMSO sets is calculated to achieve full diagnosability for all the faults in F_i. The procedure to compute 'good' compound FMSO sets starting with φ^* as a root FMSO set makes use of an optimization heuristic based on the number of shared variables and on the number of subsystems involved with the aim of minimizing communication between subsystems.

Algorithm 1. Offline Generation of LDs.

```
for i=1...n do
    Φ_i = ∅;
    Φ_i^l ← Calculate local FMSO sets of Σ_i;
    if there is any fault f ∈ F_i not locally detectable
        or not locally isolable with the set of local
        FMSO sets Φ_i^l then
        Φ_i^s ← Calculate shared FMSO sets of Σ_i;
        Λ_i^s ← Calculate shared CMSO sets of Σ_i;
    end if
    while it exists f ∈ F_i that is not detectable
        or isolable do
        Let φ* ∈ Φ_i^s such that f ∈ F_φ* be the 'best'
        (not already selected) shared FMSO set of Φ_i^s;
        Label φ* as root FMSO set: φ_r ← φ*;
        Let X^s_φr be the set of shared variables of φ_r;
        Φ_i^c* ← Build a 'good' compound FMSO set
        including φ* by always selecting the 'best'
        shared FMSO sets to cover newly introduced
        shared variables;
        Φ_i ← Φ_i ∪ Φ_i^c*;
        Φ_i^l* ← Find a minimal cardinality set of local
        FMSO sets achieving the same diagnosability
        as all local FMSO sets;
        Φ_i ← Φ_i ∪ Φ_i^l*;
    end while
end for
```

4.2 On-line distributed residual operation of LDs

After the off-line design of the LDs performed with algorithm 1, the **online** operation of the distributed diagnoser relies on the bank of residual generators ARR_i selected for each LD $LD_i, i = 1, \ldots, n$, fed by measured signals from their corresponding subsystems. As shown in Figure 3, fault isolation is carried out after fault detection using local fault signature matrices according to Definition 17.

Definition 17. (FSM of a subsystem). Given a set ARR_i composed of n_i^r ARRs and F_i the set of considered n_i^f faults for the subsystem Σ_i and consider the function $ARR_i \times F_{j,i} \longrightarrow 0, 1$, then the signature of a fault $f \in F_i$ is the binary vector $FS_i(f) = [\tau_1, \tau_2, ...\tau_{n_i^r}]^T$ where $\tau_k = 1$ if f is involved in the equations used to form $arr_k \in ARR_i$, otherwise $\tau_k = 0$. The signatures of all the faults in F_i together constitute the fault signature matrix (FSM) FSM_i for subsystem Σ_i, i.e. $FSM_i = [FS_i(f_1), \ldots, FS_i(f_{n_i^f})]^T$.

5. APPLICATION TO THE FLOTATION PROCESS

5.1 Offline distributed generation of LDs

In this section, the construction of the LD for each subsystem is presented in order to diagnose all system faults. Below the steps of the offline design:
1.- The local FMSOs are calculated for each of the subsystems, considering only local information.

$$\Phi_1^l = \Phi_2^l = \Phi_3^l = \Phi_4^l = \Phi_5^l = \emptyset \tag{2}$$

No local FMSOs were found considering only information from each subsystem. The shared FMSOs for each

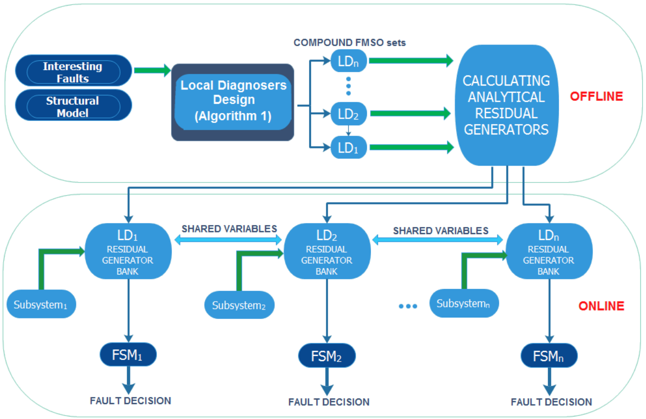

Fig. 3. Scheme of distributed generation of LDs.

Algorithm 2. On-line Residual Operation of LDs.

for i=1...n **do**
  For each LD:
  Compute ARRs for LD_i
  for j=1...m **do**
    For all selected compound FMSO sets:
    $ARR_{i,j} \leftarrow$ Compute analytical residual
    generators of LD_i;
    Save the set of known variables of
    each $ARR_{j,i}$;
    $Z_{LD_i} \leftarrow Z_{LD_i} \cup Z_{ARR_{j,i}}$;
  end for
  By means of the fault signature matrix (FSM_i)
  verify the isolability of faults of each subsystem;
end for
Add the known variables of the vector Z_{LD_i} to the fault diagnosis software for the online calculation of the ARRs of the LDs;
Generate a on-line scalar signal arr_k from
the respective $ARR_{j,i}$ using the signals of Z_{LD_i} .

subsystem are then determined by considering the vector of shared variables ($X^s = \{x_2, x_4, x_6, x_8, q_1, q_3, q_5, q_7\}$) as part of the vector of known variables for each subsystem.

2.- For subsystems σ_1 to σ_5, shared FMSO sets are computed, Results are given in Table 3.

3.- For each subsystem, Algorithm 1 chooses from the set of shared FMSO sets, a subset that is labeled as *root FMSO set* and complete with a shared FMSO set each of its shared variables until get a set of *compound FMSO sets* that can diagnose all the faults of that subsystem. The set of *compound FMSO sets* capable of detecting and isolating the faults constitute the LD of the corresponding subsystem. Results are given in Table 3.

5.2 On-line distributed residual operation of LDs

Using Algorithm 2, the ARRs are calculated and the isolation of the 16 faults of this system is verified, as shown in Table 4 to 8. As example, Figure 4 shows the ARRs operating online for subsystem 1, as can be seen in the case of a momentary fault of the tank level sensor 1 (f_1) from 600 s. up to 650 s., there is a detection of ARR_1 and no detection of ARR_2, which demonstrates the isolation of this fault locally.

Σ_1	$\Phi_1^s = \{\varphi_1, \varphi_2, \varphi_3\}$ $\varphi_1 = \{e_2, e_5, e_6\}$, $\varphi_2 = \{e_1, e_3, e_4, e_5\}$ $\varphi_3 = \{e_1, e_2, e_3, e_4, e_6\}$
Σ_2	$\Phi_2^s = \{\varphi_4, \varphi_5, \varphi_6, \varphi_7, \varphi_8, \varphi_9, \varphi_{10}, \varphi_{11}\}$ $\varphi_4 = \{e_{12}, e_{14}, e_{15}\}$, $\varphi_5 = \{e_9, e_{11}, e_{13}, e_{14}\}$ $\varphi_6 = \{e_9, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}\}$, $\varphi_7 = \{e_8, e_{10}, e_{11}, e_{13}, e_{14}\}$ $\varphi_8 = \{e_8, e_{10}, e_{11}, e_{12}, e_{13}, e_{15}\}$, $\varphi_9 = \{e_8, e_9, e_{10}, e_{14}\}$ $\varphi_{10} = \{e_8, e_9, e_{10}, e_{12}, e_{15}\}$, $\varphi_{11} = \{e_8, e_9, e_{10}, e_{11}, e_{13}\}$
Σ_3	$\Phi_3^s = \{\varphi_{12}, \varphi_{13}, \varphi_{14}, \varphi_{15}, \varphi_{16}, \varphi_{17}, \varphi_{18}, \varphi_{19}\}$ $\varphi_{12} = \{e_{21}, e_{23}, e_{24}\}$, $\varphi_{13} = \{e_{18}, e_{20}, e_{22}, e_{23}\}$ $\varphi_{14} = \{e_{18}, e_{20}, e_{21}, e_{22}, e_{24}\}$, $\varphi_{15} = \{e_{17}, e_{19}, e_{20}, e_{22}, e_{23}\}$ $\varphi_{16} = \{e_{17}, e_{19}, e_{20}, e_{21}, e_{22}, e_{24}\}$, $\varphi_{17} = \{e_{17}, e_{18}, e_{19}, e_{23}\}$ $\varphi_{18} = \{e_{17}, e_{18}, e_{19}, e_{21}, e_{24}\}$, $\varphi_{19} = \{e_{17}, e_{18}, e_{19}, e_{20}, e_{22}\}$
Σ_4	$\Phi_4^s = \{\varphi_{20}, \varphi_{21}, \varphi_{22}, \varphi_{23}, \varphi_{24}, \varphi_{25}, \varphi_{26}, \varphi_{27}\}$ $\varphi_{20} = \{e_{30}, e_{32}, e_{33}\}$, $\varphi_{21} = \{e_{27}, e_{29}, e_{31}, e_{32}\}$ $\varphi_{22} = \{e_{27}, e_{29}, e_{30}, e_{31}, e_{33}\}$, $\varphi_{23} = \{e_{26}, e_{28}, e_{29}, e_{31}, e_{32}\}$ $\varphi_{24} = \{e_{26}, e_{28}, e_{29}, e_{30}, e_{31}, e_{33}\}$, $\varphi_{25} = \{e_{26}, e_{27}, e_{28}, e_{32}\}$ $\varphi_{26} = \{e_{26}, e_{27}, e_{28}, e_{30}, e_{33}\}$, $\varphi_{27} = \{e_{26}, e_{27}, e_{28}, e_{29}, e_{31}\}$
Σ_5	$\Phi_5^s = \{\varphi_{28}, \varphi_{29}, \varphi_{30}\}$ $\varphi_{28} = \{e_{36}, e_{39}, e_{40}\}$, $\varphi_{29} = \{e_{35}, e_{37}, e_{40}\}$ $\varphi_{30} = \{e_{35}, e_{36}, e_{37}, e_{39}\}$

Table 2. Shared FMSO sets of Σ_1 to Σ_5.

LD_1	$\varphi_{31} =$	$\{e_1, e_2, e_3, e_4, e_5, e_6, e_8, e_9, e_{10}, e_{14}\}$
	$\varphi_{32} =$	$\{e_2, e_5, e_6, e_8, e_9, e_{10}, ..., e_{15}, e_{17}, e_{18}, e_{19}, e_{23}\}$
LD_2	$\varphi_{33} =$	$\{e_1, ..., e_6, e_9 e_{11} e_{13}, e_{17}, ..., e_{24}, e_{26}, e_{27}, e_{28}, e_{32}\}$
	$\varphi_{34} =$	$\{e_1, ..., e_6, e_8, e_{10}, e_{11}, e_{13}, e_{14} e_{17}, ..., e_{22}, e_{23}, e_{24}, e_{26}, e_{27}, e_{28}, e_{32}\}$
LD_3	$\varphi_{35} =$	$\{e_2, e_5, e_6, e_8, ..., e_{15}, e_{18}, e_{20}, e_{22}, e_{23}, e_{26}, ..., e_{33}, e_{38}\}$
	$\varphi_{36} =$	$\{e_2, e_5, e_6, e_8, ..., e_{15}, e_{17}, e_{19}, e_{20}, e_{22}, e_{23}, e_{26}, ..., e_{33}, e_{38}\}$
LD_4	$\varphi_{37} =$	$\{e_{12}, e_{14}, e_{15}, e_{17}, ..., e_{24}, e_{27}, e_{29}, e_{31}, e_{32}, e_{35}, e_{37}, e_{38}, e_{40}\}$
	$\varphi_{38} =$	$\{e_{12}, e_{14}, e_{15}, e_{17}, ..., e_{24}, e_{26}, e_{28}, e_{29}, e_{31}, e_{32}, e_{35}, e_{37}, e_{38}, e_{40}\}$
LD_5	$\varphi_{39} =$	$\{e_{30}, e_{32}, e_{33}, e_{35}, e_{37}, e_{38}, e_{40}\}$
	$\varphi_{40} =$	$\{e_{36}, e_{38}, e_{39}, e_{40}\}$

Table 3. Compound FMSO sets of LDs.

	Faults	
	f_1	f_2
$arr_1 \in ARR_{1,1}$	X	
$arr_2 \in ARR_{1,2}$		X

Table 4. isolation capability for ARRs for LD_1.

	Faults			
	f_3	f_4	f_5	f_6
$arr_3 \in ARR_{2,1}$		X	X	
$arr_4 \in ARR_{2,2}$	X		X	

Table 5. isolation capability for ARRs for LD_2.

	Faults			
	f_7	f_8	f_9	f_{10}
$arr_5 \in ARR_{3,1}$		X	X	
$arr_6 \in ARR_{3,2}$	X		X	

Table 6. isolation capability for ARRs for LD_3.

Finally, Figure 5 shows the human machine interface of the fault diagnosis software running on-line where a fault alarm is shown in valve 2 (f_6). This software is executed

	Faults			
	f_{11}	f_{12}	f_{13}	f_{14}
$arr_7 \in ARR_{4,1}$		X	X	
$arr_8 \in ARR_{4,2}$	X		X	

Table 7. isolation capability for ARRs for LD_4.

	Faults	
	f_{15}	f_{16}
$arr_9 \in ARR_{5,1}$	X	
$arr_{10} \in ARR_{5,2}$		X

Table 8. isolation capability for ARRs for LD_5.

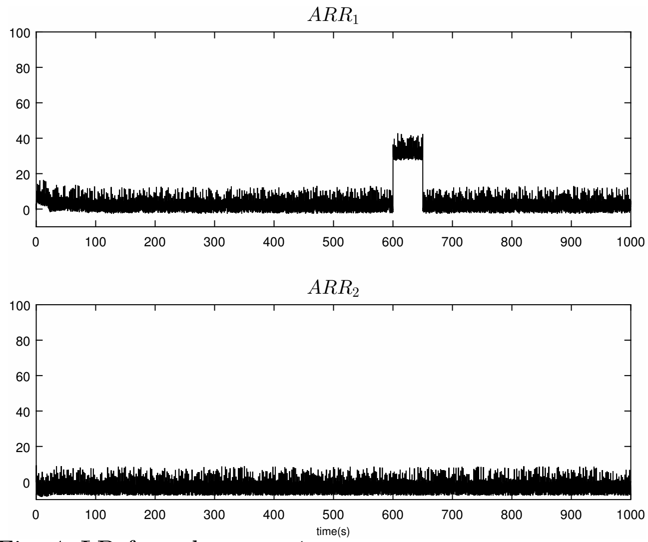

Fig. 4. LD for subsystem 1

in a programmable automation controller (PAC) that receives the signals from the sensors and generates the control signals.

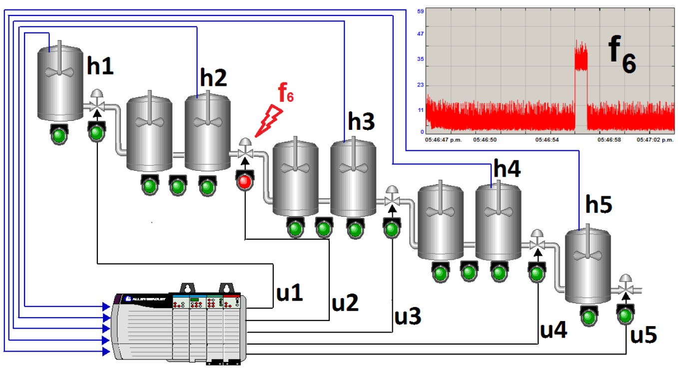

Fig. 5. Fault diagnosis software

In fact, the proposed approach is applicable to fault diagnosis of a large floating circuit, decomposing the latter into subsystems. Here, as shown above, each subsystem in the distributed architecture will have its own LD.

6. CONCLUSION

An approach for on-line fault diagnosis in a flotation process was proposed based on a distributed architecture. The application of the approach allows the development of diagnosis systems for large-scale flotation circuits. The fault diagnosis system developed, was tested by simulation validating that the 16 faults can be detected and isolated locally or at a higher level. Likewise, a procedure for residual generation was presented and it has been tested into a programmable automation controller for on-line operation of fault diagnosis software.

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