Robust hybrid control law for a boost inverter

Carolina Albea-Sanchez, Germain Garcia

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Carolina Albea-Sanchez[a,*], Germain Garciat[a]

[a]*LAAS-CNRS, Université de Toulouse, CNRS, UPS, 7 avenue du Colonel Roche, 31031 Toulouse, France*

Abstract

In this paper, we propose a robust hybrid control for a boost inverter. This control presents the particularity of considering the real nature of the dynamics, which means, the continuous-time dynamics, voltages and currents, and the discrete-time dynamics, the switching of the transistors. Likewise, due to implementation issues a minimum dwell time is introduced in the control loop scheme. Uniform global asymptotic stability is guaranteed, ensuring that the tracking error of the system output enters into a small neighbourhood of zero. Moreover, robustness guarantee with respect to parameter variations, in terms of an error tracking output regulation, are also provided here. The effectiveness of the proposed hybrid control scheme is illustrated in simulation.

Keywords: Hybrid control application, boot inverter, dwell time guarantees, robust control.

1. Introduction

In many electronic applications, it is necessary to convert a DC signal, into an AC one. Moreover, it also can be required to boost this signal, as can be found in AC microgrids, for instance. Often, it is made this transformation in two steps. Hence, in a first step, it is used an elevator DC-DC converter, and in a second one, a DC-AC converter [1, 2].

The boost inverter proposed by [3] is a particular power converter, that can generate a larger AC signal from a given DC one just in one step, with a

*Corresponding author

simple architecture. Nevertheless, to design control laws for this converter is a challenge, because it is composed by two DC-DC boost converters, being a nonlinear non-minimum phase of fourth order [4]. Furthermore, the control goal of this boost inverter is to track a sinusoidal signal.

Some control strategies have been proposed by controlling separately each DC-DC boost converter [3, 5], in such a way to generate a sinusoidal output voltage and lager than the input one. The used control techniques are PI controllers [6], sliding mode controllers [7, 8]. However, stability properties of the system in a whole is not proven.

There have been several attempts to control this inverter, considering the complete system and with the advantage of using only one output voltage reference. Hence, the authors, in [9], design a single sliding mode controller to generate the required sinusoidal voltage on the load. Moreover, they do not guarantee an error tracking output regulation in the sense of parameter variations. In [10] a control law is proposed by using energy-shaping methodology. Moreover, an adaptive controller is added to guarantee the tracking error with respect to parameter variations. However, this method is focussed on an approximated average model in continuous-time and the control law presents a relative computational cost. In [11] a cascade control focussed on sliding-mode methodology is presented. The internal control loop is based on a switching surface, generated from the difference between the inductor currents, whereas the external control loop consists of a PI compensator in order to reduce the tracking error of the inverter output voltage. Nevertheless, the authors do not consider a minimum dwell time in the control updates, which is required in implementation issues.

In this paper, a novel control law based on the hybrid dynamical system paradigm given in [12], is proposed for the considered inverter. The main advantage of this approach is to take into account the real nature of the dynamics, meaning the continuous-time dynamics, which are voltages and currents, and the discrete-time ones, which are switching signals. The control law is based on a Lyapunov matrix-based min-projection control, as used in [13, 14, 15].

One of the challenge here is to know the entire reference state corresponding to the tracking signal, which is not a simple task. To deal with this objective, we assume that the variation of the desired reference with respect to time is sufficiently slow, in order to obtain a small enough tracking error. Moreover, we consider a minimum dwell time in the hybrid control scheme, by introducing a time regulation [16]. Stability properties of the tracking error in a small neighbourhood of zero are guaranteed, applying the theory for hybrid system presented in [12]. In a second time, we propose a robust hybrid control, whit respect to parameter variations, in such a way, the system output rejects any perturbation. Some validations are performed in simulation.

This paper is organized as follows. Section 2 is devoted to the problem formulation. Some assumptions and properties are stated in Section 3. Section 4 presents the main result about the proposed hybrid control and an extension to a robust hybrid control is given in Section 5. Some simulations are performed in Section 6. Finally, the paper is closed with a conclusion section.

Notation: Through out the paper $\mathbb{N}$ denotes the set of the natural numbers and $\mathbb{R}$ the set of real numbers, $\mathbb{R}^n$ the n-dimensional euclidean space and $\mathbb{R}^{n\times m}$ the set of all real $n\times m$ matrices. The set of non-negative real numbers is denoted by $\mathbb{R}_{\geq 0}$. $M \succ 0$ (resp. $M \prec 0$) represents that M is a symmetric positive (resp. negative) definite matrix. $0_{n\times m}$ is a zero matrix of $n \times m$-dimension. $\lambda_m(M)$ and $\lambda_M(M)$ represent the minimum and maximum eigenvalues of M. $\|\cdot\|$ represents the norm euclidean.

Conflict of interest - none declared.

2. Boost inverter

The boost inverter depicted in Fig 1, is a device that can generate, in a single stage, a sinusoidal voltage with a larger amplitude than its input DC voltage, V_{in}. This converter is composed by a load R_0 differentially connected to two DC-DC converters.

The inductance currents flowing through the inductors L_1, L_2 are i_1 and i_2.

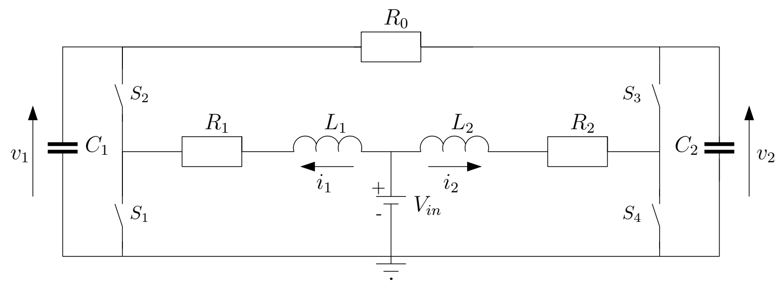

Figure 1: Boost inverter circuit.

Likewise, the capacitor voltages applied to the capacitors C_1, C_2 are v_1 and v_2. The switches are $S_i \in \{0,1\}$, with $i = \{1,2,3,4\}$, and they take the value 1, if they are switched ON, and the value 0, if they are switched OFF, respectively. Finally, R_1 and R_2 are parasitic resistances.

2.1. A Nonlinear model

The boost inverter can be represented by the following nonlinear model:

$$\begin{cases} \dot{z} = \begin{bmatrix} -\frac{R_1}{L_1} & -\frac{u}{L_1} & 0 & 0 \\ \frac{u}{C_1} & -\frac{1}{R_0 C_1} & 0 & \frac{1}{R_0 C_1} \\ 0 & 0 & -\frac{R_2}{L_2} & -\frac{1-u}{L_2} \\ 0 & \frac{1}{R_0 C_2} & \frac{1-u}{C_2} & -\frac{1}{R_0 C_2} \end{bmatrix} z + \begin{bmatrix} \frac{V_{in}}{L_1} \\ 0 \\ \frac{V_{in}}{L_2} \\ 0 \end{bmatrix} \\ w = [0 \quad 1 \quad 0 \quad -1] z, \end{cases} \tag{1}$$

where $z = [i_1 \;\; v_1 \;\; i_2 \;\; v_2]^\top \in \mathbb{R}^4$ is the state vector, $w = v_1 - v_2$ is the controlled output and $u \in \{0,1\}$ is the control input. z, is composed of continuous-time variables, whereas, u is a switching signal, thus, a discrete-time variable.

Assumption 1. *Let us consider*

- $R = R_1 = R_2$,
- $C_0 = C_1 = C_2$
- $L = L_1 = L_2$ *and*
- *the state values of the switches given in Table 1.*

u	S_1	S_2	S_3	S_4
1	1	0	1	0
0	0	1	0	1

Table 1: Boost inverter switching logic.

From Assumption 1 and the following variable changes $v := 2u - 1$ and $x= [x_1\ x_2\ x_3\ x_4]^\top = [i_1+i_2\ \ v_1+v_2\ \ i_1-i_2\ \ v_2-v_1]^\top$, the model (1) can be rewritten as:

$$\begin{cases} \dot{x} = \overbrace{\begin{bmatrix} -\frac{R}{L} & -\frac{1}{2L} & 0 & \frac{v}{2L} \\ \frac{1}{2C_0} & 0 & -\frac{v}{2C_0} & 0 \\ 0 & \frac{v}{2L} & -\frac{R}{L} & -\frac{1}{2L} \\ -\frac{v}{2C_0} & 0 & \frac{1}{2C_0} & -\frac{2}{R_0 C_0} \end{bmatrix}}^{A_v} x + \overbrace{\begin{bmatrix} \frac{2}{L} \\ 0 \\ 0 \\ 0 \end{bmatrix}}^{\mathcal{B}} V_{in} \\ y = \overbrace{[0\ 0\ 0\ 1]}^{C} x, \end{cases} \tag{2}$$

where $v \in \{-1, 1\}$ is the control variable.

The objective of this inverter is to guarantee that the output signal $y(t)$, corresponding to $v_2(t) - v_1(t)$, follows a sinusoidal reference defined by

$$V_r(t) = V_{\max} \sin(\omega t), \tag{3}$$

where $V_{\max}$ and ω are the amplitude and the angular frequency of the desired sinusoidal signal. Moreover, it is defined $w = \frac{2\pi}{T_e}$ with $T_e \in \mathbb{R}_{\geq 0}$ the time period of the desired signal. In order to impose a such behaviour to system (2), let us to introduce its averaged model, which is

$$\begin{cases} \dot{x}_a(t) = \overbrace{\begin{bmatrix} -\frac{R}{L} & -\frac{1}{2L} & 0 & \frac{2\lambda(t)-1}{2L} \\ \frac{1}{2C_0} & 0 & -\frac{2\lambda(t)-1}{2C_0} & 0 \\ 0 & \frac{2\lambda(t)-1}{2L} & -\frac{R}{L} & -\frac{1}{2L} \\ -\frac{2\lambda(t)-1}{2C_0} & 0 & \frac{1}{2C_0} & -\frac{2}{R_0 C} \end{bmatrix}}^{A_{\lambda(t)}} x_a(t) + \overbrace{\begin{bmatrix} \frac{2}{L} \\ 0 \\ 0 \\ 0 \end{bmatrix}}^{\mathcal{B}} V_{in}, \\ y_a(t) = [0\ 0\ 0\ 1] x_a(t), \end{cases} \tag{4}$$

where $x_a(t)$ represents the averaged signal of x and parameter $\lambda(t) \in [0, 1]$ is the duty cycle of v. This averaged model allows to define the desired complete reference of state x associated with $V_r(t)$, i.e., the ideal reference associated with infinity switching.

One can see, the time-varying reference of (2) is given considering the desired output reference (3) for $y_a(t)$ of system (4) and its associated variable $0 \leq \lambda_e(t) \leq 1$, such that,

$$x_{ae}(t) = -(\lambda_e(t)A_1 + (1 - \lambda_e(t))A_{-1})^{-1}\mathcal{B} \tag{5}$$

$$y_{ae}(t) = V_r(t) \tag{6}$$

is the equilibrium of (4). This desired reference obtained satisfying (5)–(6) is perceived as relaxed solutions in the generalized sense of Filippov. Nevertheless, to obtain an explicit expression of $x_{ae}(t)$ is not a trivial task.

From the introduced boost inverter model (2) and its reference (5)–(6), we are in condition of stating the problem.

Problem 1. *The first goal in this work is to find a tracking reference of* (2)*, $x_e(t)$, such that, $\|x_e(t) - x_{ae}(t)\|$ is bounded and $\|y_e(x_e(t)) - V_r(t)\|$ is small enough. A second goal deals with designing a hybrid control law for system* (2)*, such that, for any initial condition $x(0) \in \mathbb{R}^4$, the following holds:*

- *$\|x(t) - x_{ae}(t)\|$ is bounded,*
- *$\lim_{t\to\infty} y(t) - V_r(t)$ presents an error small enough,*
- *the output is robust with respect to parameter variations and modelled errors.*

3. Preliminaries

The following property is a standard property found in the literature [13, 17, 15], that characterizes the equilibrium in switched affine systems.

Property 1. *Given matrices A_1 and A_{-1} in model* (2)*, a matrix $Q \succ 0 \in \mathbb{R}^4$, there exists $\lambda \in \Lambda$ and a matrix $P \succ 0 \in \mathbb{R}^4$ satisfying*

$$A_i^\top P + PA_i + 2Q \prec 0, \quad \textit{for} \;\; i = \{-1, 1\}. \tag{7}$$

Note that Property 1 is direct if A_{-1} and A_1 are Hurwitz.

This property allows introducing the next definition. The averaged model of (2) is given by

3.1. Operation point definition

Let us consider the particular case, corresponding to a constant V_r, i.e., $\omega = 0$. Then, the desired state reference generated by (5) is

$$\dot{x}_{ae}(t) = 0_{4\times 1}, \qquad y_{ae} = V_r. \tag{8}$$

In this particular case, we can formulate the following assumption for the desired operating point of (2).

Assumption 2. *Consider the desired output reference* (8) *and a parameter* $0 \leq \lambda_e \leq 1$*, such that,*

$$x_e = -(\lambda_e A_1 + (1 - \lambda_e)A_{-1})^{-1}\mathcal{B}$$

is the equilibrium point of (4)*.*

This equilibrium is also the equilibrium of system (2) related to arbitrarily fast switching. It is worth noting, if Assumption 2 is satisfied and from Property 1, it is easy to see that $(1 - \lambda_e)(A_{-1}x_e + \mathcal{B}) + \lambda_e(A_1 x_e + \mathcal{B}) = \mathbf{0}$.

Proposition 1. *Consider that R is negligible (i.e. $R \ll R_0$), Assumption 1 and Assumption 2 are satisfied, then*

i) the equilibrium point can be expressed as

$$x_e(\lambda_e) = \begin{bmatrix} \frac{(2\lambda_e - 1)^2}{\lambda_e^2 R_0 (\lambda_e - 1)^2} \\ \frac{-1}{\lambda_e(\lambda_e - 1)} \\ \frac{(2\lambda_e - 1)}{\lambda_e^2 R_0 (\lambda_e - 1)^2} \\ \frac{-(2\lambda_e - 1)}{\lambda_e(\lambda_e - 1)} \end{bmatrix} V_{in}, \qquad 0 < \lambda_e < 1. \tag{9}$$

ii) The reference of the constant output voltage, V_r, provides (given also in [3])

$$\frac{V_r}{V_{in}} = -\frac{2\lambda_e - 1}{\lambda_e(\lambda_e - 1)}.$$

$$V_r \lambda_e^2 + (2V_{in} - V_r)\lambda_e - V_{in} = 0,$$

and their roots are

$$\lambda_e(V_r) = \begin{cases} \frac{1}{2} - \frac{V_{in}}{V_r} + \sqrt{\frac{1}{4} + \frac{V_{in}^2}{V_r^2}} & \text{if} \quad V_r \geq 0 \\ \frac{1}{2} & \text{if} \quad V_r = 0 \\ \frac{1}{2} - \frac{V_{in}}{V_r} - \sqrt{\frac{1}{4} + \frac{V_{in}^2}{V_r^2}} & \text{if} \quad V_r \leq 0. \end{cases} \tag{10}$$

Therefore, for a given value of V_r, it is possible, from the previous formulation, to determine the corresponding value of $\lambda_e(V_r)$, inducing the equilibrium point x_e of (2).

3.2. Property of the sinusoidal reference signal

Now, we consider the sinusoidal reference (3) that generates the reference dynamic described in (5)–(6), with ω different to zero.

The next Lemma will be devoted to prove that $\lambda_e(V_r(t))$ given in (10) is a periodic function, with the same time period than the reference signal $V_r(t)$.

Lemma 1. *We have the following properties:*

i) The function $t \mapsto \lambda_e(V_r(t))$ is a continuous and differentiable function from $\mathbb{R}$ to $]0, 1[$.

ii) If the function $V_r(t)$ is a periodic function of period T_e, then the function $t \mapsto \lambda_e(V_r(t))$ is a periodic function of period T_e, with a mean value equal to $1/2$.

Proof. From the function given in (10), we have

$$\lim_{V_r \to \infty} \lambda_e(V_r) = 1 \quad \text{and} \quad \lim_{V_r \to -\infty} \lambda_e(V_r) = 0.$$

The continuity and differentiability is obvious for all $V_r \in \mathbb{R}$. We have from (10)

$$\lim_{V_r \to 0} \lambda_e(V_r) = \lim_{V_r \to 0} \left(\frac{1}{2} - \frac{V_{in}}{V_r} \pm \left| \frac{V_{in}}{V_r} \right| \right).$$

Then, we can deduce

$$\lim_{V_r \to 0^+} \lambda_e(V_r) = \lim_{V_r \to 0^-} \lambda_e(V_r) = \frac{1}{2}$$

and, therefore $\lambda_e(V_r)$ is continuous at $V_r = 0$. A simple calculation leads also to

$$\frac{d\lambda_e}{dV_r} = \begin{cases} \frac{V_{in}}{V_r^2} - \frac{V_{in}^2}{V_r^3 \sqrt{\frac{1}{4} + \frac{V_{in}^2}{V_r^2}}} & \text{if} \quad V_r > 0 \\ \frac{V_{in}}{V_r^2} + \frac{V_{in}^2}{V_r^3 \sqrt{\frac{1}{4} + \frac{V_{in}^2}{V_r^2}}} & \text{if} \quad V_r < 0. \end{cases}$$

Moreover,

$$\lim_{V_r \to 0} \frac{d\lambda_e}{dV_r} = \lim_{V_r \to 0} \left(\frac{V_{in}}{V_r^2} \pm |V_r| \frac{V_{in}}{V_r^3} \right),$$

then,

$$\lim_{V_r \to 0^+} \frac{d\lambda_e}{dV_r} = \lim_{V_r \to 0^-} \frac{d\lambda_e}{dV_r} = 0.$$

Thus, $\lambda_e(V_r)$ is differentiable at $V_r = 0$. Simple calculations show that $\lambda_e(V_r)$ presents an inflection point at $V_r = 0$, noting

$$\frac{d\lambda_e}{dV_r} > 0 \ \text{ if } \ V_r > 0 \ \text{ and } \ \frac{d\lambda_e}{dV_r} > 0 \ \text{ if } \ V_r < 0.$$

The proof of i) is complete. Then, the periodicity of $\lambda_e(V_r(t))$ follows by continuity arguments and the fact that

$$\frac{d\lambda_e(V_r(t))}{dt} = \begin{cases} \left[\frac{V_{in}}{V_r^2} - \frac{V_{in}^2}{V_r^3 \sqrt{\frac{1}{4} + \frac{V_{in}^2}{V_r^2}}} \right] \frac{dV_r}{dt} & \text{if} \quad V_r > 0 \\ \left[\frac{V_{in}}{V_r^2} + \frac{V_{in}^2}{V_r^3 \sqrt{\frac{1}{4} + \frac{V_{in}^2}{V_r^2}}} \right] \frac{dV_r}{dt} & \text{if} \quad V_r < 0. \end{cases} \tag{11}$$

Furthermore, note that the mean of $V_r(t)$ is 0, hence the mean of $\lambda_e(V_r(t)) = \frac{1}{2}$, concluding the proof of item ii). $\square$

Remark 1. It is worth noting Lemma 1 allows to obtain an approximative expression of the periodic equilibrium of $x_{ae}(t)$, such that, $x_e(\lambda_e(V_r(t))) \to x_{ae}(t)$, as $\frac{dx_{ae}(t)}{dt} \to 0$. ⌟

4. An hybrid control Scheme

The inverter model presented in (2) covers two different types of dynamics. In one hand, the continuous-time variables which are the differences and the sums of the inductor currents and the capacitor voltages. In other hand, the control input, v, is a discrete-time variable. Consequently, the framework given in [12] about hybrid dynamical system is well suited.

The main idea of dealing with Problem 1 is to use Proposition 1 to generate a time-periodic reference for $x(t)$, from $V_r(t)$ given in (3), as noted in Remark 1. Moreover, if the variation of $x_e(\lambda_e)$ with respect to time is sufficiently slow, we can expect that the output $y(t) = x_4(t)$ will be closed to the desired reference $V_r(t)$ given in (3).

This basic idea is illustrated by the bloc scheme represented in Fig. 2.

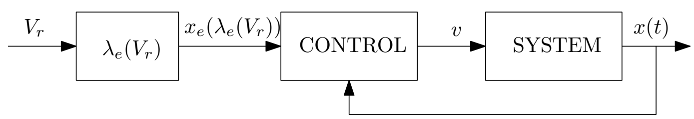

Figure 2: Block diagram of the control mechanism.

It is easy to see that the generated output of this control process, $y(t)$, will present an error in steady state from the desired reference, $V_r(t)$. Indeed, for one side, there is an error between the output generated from the averaged model $y_{ae} = V_r(t)$ and the one generated from the computed reference $y_e := [0\ 0\ 0\ 1]x_e(\lambda_e)$, given in Proposition 1. For another side, due to implementation issues, it must have a minimum dwell time in the control updates, inducing an error in steady state between $x(t)$ and the provided reference $x_e(\lambda_e)$.

4.1. Hybrid scheme

As proposed in [15], a new state variable τ is added to ensure a minimum positive dwell time, T. Then, the closed loop system is proposed in the hybrid formulation $\mathcal{H}$ given below.

4.1.1. Hybrid model

$$\mathcal{H}:\begin{cases}\begin{bmatrix}\dot{\tilde{x}}\\ \dot{v}\\ \dot{\tau}\\ \dot{x}_{ae}\\ \dot{\lambda}_e\\ \dot{x}_e\end{bmatrix}=\begin{bmatrix}A_v\tilde{x}+B_v\\ 0\\ 1-dz(\frac{\tau}{T})\\ A_{\lambda_e}x_{ae}+\mathcal{B}\\ f_1(V_r)\\ f_2(\lambda_e)\end{bmatrix}, & \zeta\in\mathcal{C}\\ \begin{bmatrix}\tilde{x}^+\\ v^+\\ \tau^+\\ x_{ae}^+\\ \lambda_e^+\\ x_e^+\end{bmatrix}\in\begin{bmatrix}x\\ \underset{i\in\{-1,1\}}{\operatorname{argmin}}\ \tilde{x}^\top P(A_i\tilde{x}+B_v)\\ 0\\ x_{ae}\\ \lambda_e\\ x_e\end{bmatrix}, & \zeta\in\mathcal{D},\\ \tilde{y}\ =\ C(\tilde{x}+x_e-x_{ae}) \end{cases} \tag{12}$$

where $\tilde{x}:=x-x_e$, $\zeta:=[\tilde{x}\ v\ \tau\ x_{ae}\ \lambda_e\ x_e]^\top\in\mathbb{H}$, such that,

$$\mathbb{H}:=\{\tilde{x}\in\mathbb{R}^4, v\in\{-1,1\}, \tau\in[0,T], x_{ae}\in\mathbb{R}^4, \lambda_e\in[\nu,1-\nu], x_e\in\mathbb{R}^4\},$$

with a parameter $\nu>0$ small enough. Likewise, $B_v:=A_vx_e+\mathcal{B}$, $f_1(V_r):=\frac{d\lambda_e(V_r(t))}{dt}$ given in (11) and $f_2(\lambda_e)$ is the derivative of (9) with respect to time. dz defines a dead-zone function, for all $s\geq 0$, $dz(s):=\max\{0,s-1\}$.

Inspired by [15], we select the so-called flow and jump sets

$$\mathcal{C}:=\{\zeta:\tilde{x}^\top P(A_v\tilde{x}+B)\leq-\tilde{x}^\top Q\tilde{x},\ \tau\in[0,T]\} \tag{13}$$

$$\mathcal{D}:=\{\zeta:\tilde{x}^\top P(A_v\tilde{x}+B)\geq-\tilde{x}^\top Q\tilde{x},\ \tau=T\}, \tag{14}$$

ensuring that all maximal solutions are complete. Note that, τ introduces a minimum dwell time, T, i.e., the solutions are forced to flow at least T ordinary time after each jump. Then, the following condition must hold:

$$T\ll T_e. \tag{15}$$

Proposition 2. *The hybrid dynamical system* (12)–(14) *satisfies the basic hybrid conditions [12, Assumption 6.5], then it is well-posed.*

Proof. The hybrid dynamical system (12)–(14) satisfies the basic hybrid conditions because

- sets $\mathcal{C}$ and $\mathcal{D}$ are closed.
- f is a continuous function, thus it is outer semicontinuous and convex. Moreover, it is locally bounded.
- G is closed, then it also is outer semicontinuous [12, Lemma 5.1] and, it is locally bounded.

Finally, following [12, Theorem 6.30], we conclude that the hybrid dynamical system is well posed. □

In order to achieve the objectives stated in Problem 7, we will provide a hybrid control scheme establishing stability properties for $x = x_e$, considering x_e constant and a minimum dwell time in the implementation. Then, we will assume the time varying of $x_e(t)$, finding a bound of $\|y_e(t) - y_{ae}(t)\|$. Finally, stability properties of $y(t) = y_{ae}(t)$, will be ensured.

4.2. Stability properties for V_r constant

The objective here is to regulate the state vector, x, to a neighbourhood of a constant point $x_e(\lambda_e(V_r))$, providing suited stability properties by using the paradigm presented in [12]. Then, for any parameter $X > 0$ and a given $T > 0$ the goal is to ensure UGAS of the compact attractor

$$\mathcal{A} := \{\zeta \in \mathbb{H} : \|x - x_e\| \leq X, \tau \in [0, T]\}.$$

To this end, we invoke the next lemma.

Lemma 2. *[16] Consider that Property 1 is satisfied, then the eigenvalues of matrix $P^{-1}Q$ are positive and $\|e^{A_v t}\| \leq \frac{\lambda_M^{1/2}}{\lambda_m^{1/2}} e^{-\alpha t}$, whit $\alpha = \lambda_m(P^{-1}Q)$.*

Moreover, we need to provide a practical minimum dwell-time property for $\mathcal{H}$ due to implementation issues. Then, next property is established.

Property 2. *There exists a positive scalar T^*, such that, for any chosen $T \in [0, T^*]$, the solutions to hybrid system $\mathcal{H}$ flow for, at least, T ordinary time after the jump, before reaching set $\mathcal{D}$.*

Proof. From the proof given in [16] and without loss of generality, we consider that the first jump occurs in $t = t_0$, defining $\tilde{t} := t - t_0$. Then, the trajectories of the error dynamics of (2), given by $\tilde{x} = x - x_e$ flowing in $\mathcal{C}$, as follows,

$$\tilde{x}(\tilde{t}) = e^{A_v \tilde{t}}(\tilde{x}_0 + A_v^{-1} B_v) - A_v^{-1} B_v.$$

From Lemma 2, we have

$$\|\tilde{x}(\tilde{t}) - A_v^{-1} B\| \leq \frac{\lambda_M^{1/2}(P)}{\lambda_m^{1/2}(P)} e^{\alpha \tilde{t}} \|\tilde{x}_0 + A_v^{-1} B_v\|.$$

Then, for all $0 \leq \tilde{t} \leq T$, we get

$$0 \leq \tilde{t} \leq \frac{1}{\alpha} \mathrm{Ln} \left(\frac{\lambda_M^{1/2}(P) \|\tilde{x}_0 + A_v^{-1} B\|}{\lambda_m^{1/2}(P) \|\tilde{x}(\tilde{t}) - A_v^{-1} B\|} \right).$$

It is worth noting that in the instant time $\tilde{t} = T$ the solution to $\mathcal{H}$ jumps, i.e., the solution is in $\mathcal{D}$. Thus, it is possible to define a dwell-time upper bound, $T^* \geq T \geq 0$, as follows,

$$T^* = \max_{i \in \{-1,1\}} \frac{1}{\alpha} \mathrm{Ln} \left(\frac{\lambda_M^{1/2}(P) \|\tilde{x}_0 + A_i^{-1} B_i\|}{\lambda_m^{1/2}(P) \|\tilde{x}(T^*) - A_i^{-1} B_i\|} \right).$$

$\square$

Note that for a maximum possible chattering given by $\tilde{x}_0$ and $\tilde{x}(T^*)$, we get the upper bound T^*.

The next Lemma is a fundamental step to prove stability results. Then, we evoke [14, Lemma 1], which is necessary to provide stability properties of (12)–(14).

Lemma 3. *Consider matrices $P, Q \in \mathcal{S}^4$ satisfying Property 1, for each point $x_e \in \mathbb{R}^4$ satisfying Assumption 2, then,*

$$\min_{i \in \{-1,1\}} \tilde{x}^\top P(A_i \tilde{x} + B) \leq -\tilde{x}^\top Q \tilde{x}, \tag{16}$$

with $\tilde{x} = x - x_e$.

Now, we are able to introduce the asymptotic stability result for the regulation problem, i.e., for a constant reference x_e.

Theorem 1. *Consider an operating point* $x_e \in \mathbb{R}^{4\times 1}$ *that satisfies Assumption 2. Moreover, consider that matrices* $P \in \mathbb{R}^4 \succ 0$ *and* $Q \in \mathbb{R}^4 \succ 0$ *satisfy Property 1. Then, for any* $X > 0$ *and a given scalar* $T \in [0, T^*]$ *the following holds:*

1. *set* $\mathcal{A}$ *is compact,*
2. *set* $\mathcal{A}$ *is UGAS for hybrid system* (12)–(14)*,*
3. *the system output is bounded by* $\|y - V_r\| < X$*,*
4. *set* $\mathcal{A}_0 := \{\zeta : \tilde{x} = \mathbf{0}, u \in \{0,1\}, \tau \in [0,T]\}$ *is globally practically asymptotically stable for system* (12)–(14)*. As long as,* T *is small enough, set* $\mathcal{A}$ *can be arbitrarly close to* $\mathcal{A}_0$*.*

Proof. First, we will prove the compactness of attractor $\mathcal{A}$.

Proof of item 1) Hybrid system (12)–(14) satisfies Property 2, then we can define an upper bound of $\tilde{x}$ in $\mathcal{C}$

$$\begin{aligned}\|\tilde{x}(t)\| &\leq \max_{i\in\{-1,1\}}\left(\frac{\lambda_M^{1/2}(P)}{\lambda_m^{1/2}(P)}e^{\alpha\tilde{t}}\|\tilde{x}(t_0) + A_i^{-1}B_i\| + \|A_i^{-1}B_i\|\right)\\ &:= X,\end{aligned}$$

such that, t_0 is any time instant where occurs a jump in the steady state. Then, we ensure that $\mathcal{A}$ is compact.

Now, from Proposition 2, it is possible apply useful well-posed hybrid results.

Proof of item 2) Let us consider the following Lyapunov function,

$$V_1(\tilde{x}) := \frac{1}{2}\tilde{x}^\top \bar{P}\tilde{x}, \tag{17}$$

with $\tilde{x} = x - x_e$. Then, from Lemma 3, it is easy to see that during flows, the solutions are in $\mathcal{C}$, i.e., in (13), therefore, the following holds

$$\langle \nabla V_1(\tilde{x}), f(\zeta)\rangle = \tilde{x}^\top P(A_v x + B_v) \leq -\tilde{x}^\top \bar{Q}\tilde{x}.$$

When a jump occurs, we get

$$V_1(\tilde{x}^+) - V_1(\tilde{x}) = 0, \tag{18}$$

since $\tilde{x}^+ = [x^+ - x_e] = \tilde{x}$.

Then, UGAS of $\mathcal{A}$ is shown from the proof given in [14, 15, Theorem 1].

Proof of item 3) The proof is direct from item 2).

Proof of item 4) From Property 2 is easy to see that as T goes to zero, the minimum dwell time, T goes also to zero. Then, from a sufficiently small T, set $\mathcal{A}$ shrinks to $\mathcal{A}_0$. □

4.3. Stability properties for $V_r(t) = V_{\max}\sin(\omega t)$

For $\lambda_e(V_r)$ fixed, the proposed hybrid control allows to globally stabilize the sampled equilibrium (9), as seen above. Now, we assume a time-varying non-sampled reference, $V_r(t)$, in particular take $\lambda_e = \lambda_e(V_{\max}\sin(\omega t))$. From the property of this function given in Proposition 1 and by continuity of (9) with respect to λ_e, we can deduce that $x_e(\lambda_e)$ is also a periodic continuous function of time. Intuitively, if the variation of x_e with respect to time is sufficiently slow, we can expect that the output $y_e(x_e)$ be close to $y_{ae} = V_{\max}\sin(\omega t)$.

Stability properties of the tracking problem of system 2 are established in next lemma.

Lemma 4. *Consider the averaged model* (4) *and $x_e = x_e(V_{\max}\sin(\omega t))$, being $x_e(\lambda_e)$ given by* (9)*, such that, we have*

$$A_{\lambda_e(V_{\max}\sin(\omega t))} x_e(t) + \mathcal{B} = 0. \tag{19}$$

Assume that there exist parameters $\rho > 0$ and $\epsilon > 0$, such that, $Q = C^\top C + \epsilon I$ and a positive definite symmetric matrix $P \succ \mathbb{R}^4$ satisfying

$$\begin{aligned} &\min_{P,\rho} \rho \\ &\quad s.t.\ \rho > 0 \\ &\qquad P > 0 \end{aligned}$$

$$\begin{bmatrix} A_v^\top P + P A_v + 2Q & -P \\ -P & -\rho I \end{bmatrix} \leq 0, \qquad v = -1, 1 \tag{20}$$

then, for each solution to

$$\begin{cases} \frac{dx_{ae}}{dt} = A_{\lambda_e(t)} x_{ae} + \mathcal{B} \\ y_{ae} \ = C x_{ae}, \end{cases} \tag{21}$$

with $x_{ae}(0) = x_e(0)$ *and* $\lambda_e(t) = \lambda_e(V_{\max} \sin(wt))$*. When* $t \to \infty$*, it holds, for* $\gamma^2 = \rho$*, that*

$$\int_t^{t+T_e} |y_e(\bar{t}) - V_{\max} \sin(\omega \bar{t})|^2 d\bar{t} \leq \gamma^2 \int_t^{t+T_e} \left\| \frac{dx_{ae}(\bar{t})}{d\bar{t}} \right\|^2 d\bar{t},$$

where $y_e(t) := C x_e(\lambda_e)$*.*

Proof. The proof is inspired by the proof of [18, Theorem 3.1]. It is important to note that if Property (1) is satisfied, then the system

$$\frac{dx_{ae}}{dt} = A_{\lambda_e(t)} x_{ae}$$

is an asymptotically stable T_e-periodic system. Moreover, it results from [19, Theorem 4.7] or [20, Theorem 1], that there exists a unique steady state periodic solution $x_{ss}(t)$ to (4). It means that

$$\lim_{t \to \infty} x_{ae}(t) \to x_{ss}(t), \tag{22}$$

where

$$x_{ss}(t + T_e) \to x_{ss}(t).$$

Consider now the Lyapunov function $V_2(x_e, x_{ae}) = \frac{1}{2}(x_e - x_{ae})^\top P (x_e - x_{ae})$, then the derivative of V_2 along the trajectories of (4) is given as:

$$\begin{aligned} \frac{dV_2}{dt} = & (x_e - x_{ae})^\top P \frac{dx_{ae}}{dt} + \frac{dx_{ae}}{dt}^\top P (x_e - x_{ae}) \\ & - (x_e - x_{ae})^\top P \frac{dx_{ae}}{dt} - \frac{dx_{ae}}{dt}^\top P (x_e - x_{ae}), \end{aligned}$$

and due to (19), we can write

$$\frac{dx_a}{dt} = A_{\lambda_e(t)} (x_e - x_{ae}).$$

Then,

$$\frac{dV_2}{dt} = (x_e - x_{ae})^\top (A_{\lambda_e(t)})^\top P + P A_{\lambda_e(t)})(x_e - x_{ae}) - 2(x_e - x_{ae})^\top P \frac{dx_{ae}}{dt}.$$

Multiplying (20) on the left by $\left[(x_e - x_{ae}) \quad \frac{dx_{ae}}{dt}\right]^\top$, using the selected $Q = C^\top C + \epsilon I$ and on the right by its transpose, we obtain

$$\begin{aligned}(x_e - x_{ae})^\top (A_v^\top P + P A_v + 2\epsilon I + 2C^\top C)(x_e - x_{ae})\\ -2(x_e - x_{ae})^\top P \frac{dx_{ae}}{dt} - \gamma^2 \frac{dx_{ae}}{dt}^\top \frac{dx_{ae}}{dt} \leq 0,\end{aligned}$$

with $v = \{-1, 1\}$. Note that $A_1 = A(1)$ and $A_{-1} = A(0)$.
Now, multiplying the first inequality by $\lambda(t)$ and the second one by $1 - \lambda(t)$, and summing the two resulting inequalities, we obtain:

$$\begin{aligned}(x_e - x_{ae})^\top (A_{\lambda_e(t)}^\top P + P A_{\lambda_e(t)} + 2\epsilon I + 2C^\top C)(x_e - x_{ae})\\ -2(x_e - x_{ae})^\top P \frac{dx_{ae}}{dt} - \gamma^2 \frac{dx_{ae}}{dt}^\top \frac{dx_{ae}}{dt} \leq 0.\end{aligned}$$

Then, by the positivity of $(x_e - x_{ae})^\top 2\epsilon I (x_e - x_{ae})$ we have

$$\frac{dV_2}{dt} + 2|y_e(t) - y_{ae}(t)|^2 - \gamma^2 \left\| \frac{dx_{ae}(t)}{dt} \right\|^2 dt \leq 0,$$

which in turns by integration on $[t, t + T_e]$ gives

$$\begin{aligned}V_2\left(x_e(t + T_e), x_{ae}(t + T_e)\right) - V_2(x_e(t), x_{ae}(t)) +\\ \int_t^{t+T_e} 2|y_e(\bar{t}) - y_{ae}(\bar{t})|^2 d\bar{t} - \gamma^2 \int_t^{t+T_e} \left| \frac{dx_{ae}(\bar{t})}{d\bar{t}} \right|^2 d\bar{t} \leq 0.\end{aligned}$$

From equation (22), we have:

$$\begin{aligned}V_2(x_e(t + T_e), x_{ss}(t + T_e)) - V_2(x_e(t), x_{ss}(t)) +\\ \int_t^{t+T_e} |y_e(\bar{t}) - y_{ae}(\bar{t})|^2 d\bar{t} - \frac{\gamma^2}{2} \int_t^{t+T_e} \left\| \frac{dx_{ae}(\bar{t})}{d\bar{t}} \right\|^2 d\bar{t} \leq 0,\end{aligned}$$

and by the periodicity of $x_{ss}(t)$,

$$\int_t^{t+T_e} |y_e(\bar{t}) - y_{ae}(\bar{t})|^2 d\bar{t} \leq \frac{\gamma^2}{2} \int_t^{t+T_e} \left\| \frac{dx_{ae}(\bar{t})}{d\bar{t}} \right\|^2 d\bar{t}. \tag{23}$$

To complete the proof, it suffices now to remark that $y_{ae}(t) = V_{\max}\sin(\omega t)$.

$\square$

From this lemma, we can provide our main result about controlling the boost inverter working as DC-AC converter.

Theorem 2. *For a given output reference* $V_r = V_{\max}\sin(\omega t)$*, consider* $x_e(\lambda_e(V_r(t)))$*, defined in Proposition 1 and some given parameters* T_e *and* T*, such that condition* (15) *is satisfied. Moreover, consider there exist matrices* $P \in \mathbb{R}^4 \succ 0$*,* $Q \in \mathbb{R}^4 \succ 0$ *and parameters* $\gamma^2 = \rho > 0$ *and* $\epsilon > 0$*, such that, all Lemma 4 assumptions are satisfied. Then, for any parameters* $X_t > 0$*,* $X > 0$ *and for*

$$\varepsilon_0(t) := \frac{\gamma^2}{2}\int_t^{t+T_e}\left\|\frac{dx_{ae}(\bar{t})}{d\bar{t}}\right\|^2 d\bar{t}.$$

and any scalar $T \in [0, T^*]$ *the following holds:*

1. *set*

$$\mathcal{A}_t := \{\zeta \in \mathbb{H} : \|x - x_e\| \leq X + X_t, \tau \in [0, T]\}.$$

   is compact,

2. *set* $\mathcal{A}_t$ *is UGAS for hybrid system* (12)–(14)*,*

3. *the system output is bounded by* $\int_t^{t+T_e}|y(t) - V_r(\bar{t})|^2 d\bar{t} < T_e X^2 + \varepsilon_0$*,* (12)–(14)*,*

4. *set* $\mathcal{A}_{t,0} := \{\zeta : \|x - x_e\| \leq X_t, u \in \{0,1\}, \tau \in [0,T]\}$ *is globally practically asymptotically stable for system* (12)–(14)*. As long as,* T *is small enough, set* $\mathcal{A}$ *can be arbitrarily close to* $\mathcal{A}_0$*.*

Proof. The proof mainly follows the proof of Theorem 1.

Proof of item 1) From the proof of Theorem 1, here, the proof is completed from Lemma 4, which provides a bound of $\|y_e - V_r(t)\|$, which implies the bound $\|x_e - x_{ae}\| \leq \psi(\varepsilon_0(t)) \leq X_t$. Then, $\|x - x_e\| \leq \|x - x_e\| + \|x_e - x_{ae}\| \leq X + X_t$.

Proof of item 2) In this item, we will consider the following Lyapunov candidate

$$V(x, x_e, x_{ae}) := V_1(x, x_e) + V_2(x_e, x_{ae}) \tag{24}$$

defined in (17) and Lemma 4 proof. It is easy to see the solution to the optimisation problem given in Lemma 4, satisfy the assumptions of Theorem 1. Then, the solution to hybrid system (12)–(14) goes to the interior of $\mathcal{A}_t$, guaranteeing that this set is UGAS.

Proof of item 3) The output error is $|y-y_{ae}|^2 = |y-V_{\max}\sin(\omega t)|^2$, integrating from t to $t+T_e$ and applying Theorem 1 and Lemma 4 to bound $|y-y_e|^2$ and $|y_e-y_{ea}|^2$ respectively, we have $\int_t^{t+T_e}|y(\bar{t})-V_r(\bar{t})|^2 d\bar{t} < \int_t^{t+T_e}|y(\bar{t})-y_e(\bar{t})|^2 d\bar{t} + \int_t^{t+T_e}|y_e(\bar{t})-V_r(\bar{t})|^2 d\bar{t} < T_e X^2 + \varepsilon_0$.

Proof of item 4) Direct from the proof of Theorem 1.

□

Remark 2. If $x_{ae}(t)$ is constant, the right term in the inequality (23) is equal to zero and $y_e(t) \to V_r(t)$. As expected, the error depends on the variations of $x_{ae}(t)$ and γ^2 can be interpreted as the rejection gain when $\frac{dx_{ae}(t)}{dt}$ is considered as a perturbation signal. ⌟

Remark 3. The approach proposed above is based on the assumption $R \ll R_0$. If this assumption is not satisfied, it is not possible to obtain an analytical expression for $\lambda_e(V_r)$. In such case, the value of $\lambda_e(V_r)$ is characterized by:

$$V_r = -\lambda_e R_0 V_{in} \frac{(2\lambda_e - 1)(\lambda_e - 1)}{\lambda_e^2 R_0(\lambda_e-1)^2 + R(2\lambda_e^2 - 2\lambda_e + 1)}.$$

Recall that only the admissible values of λ_e are real, meaning $0 < \lambda_e < 1$. The previous equation can also be written as

$$1 + \frac{V_r}{R_0 V_{in}} \frac{\lambda_e^2 R_0(\lambda_e-1)^2 + R(2\lambda_e^2 - 2\lambda_e + 1)}{\lambda_e(2\lambda_e - 1)(\lambda_e - 1)} = 0.$$

Invoking the simple root locus building rules, we can easily see that depending of the sign of V_r, only one root is of interest and varies with V_r around $1/2$. Then it is possible to compute the value of λ_e for a given value of V_r. All the values of λ_e for a given function $V_r(t)$ can be determined off-line. The approach developed above can then be applied. ⌟

It is worth noting that Theorem 2 provides a solution for almost all items stated in Problem 1. However, the last item related with robustness is not still considered.

5. Robustness

In many occasions, these electronic systems suffer variations in the load, R, in the voltage input, V_{in}, or in other components, needing to guaranty output robustness with respect to parameter variations, or even, to modelled errors. To this deal, we introduce an extra dynamic to regulate the voltage output, $y(t)$. This dynamic is given by

$$\dot{\varrho} = -K_1\varrho - K_1K_2\tilde{x}_4, \tag{25}$$

where K_1 and K_2 are predefined positive parameters, such that, Property 1 holds. The variable ϱ gathers the current reference variations to compensate any perturbation occurred in the voltage due to any parameter change. Therefore, it is needed to add ϱ to the current references. In particular, to the reference of $x_{e,1} = i_1 + i_2$, and $x_{e,3} = i_1 - i_2$. Thus, we can define

$$x_{e,\varrho} := x_e(\lambda_e) - \Upsilon\varrho,$$

where $x_e(\lambda_e)$ is given in Property 1 and $\Upsilon := [1 \quad 0 \ 1 \quad 0]^\top$. Defining the augmented vectors $\chi := [x \quad \varrho]^\top$, $\chi_{ae} := [x_{ea} \quad 0]^\top$ and $\chi_e := [x_{e,\varrho} \quad 0]^\top$, as well as, $\tilde{\chi} := \chi - \chi_e$, we get the following error system dynamics

$$\begin{cases} \dot{\tilde{\chi}} = \bar{A}_v\tilde{\chi} + \bar{B}_v, \\ \tilde{y} = \bar{C}(\tilde{\chi} + \chi_e - \chi_{ae}), \end{cases} \tag{26}$$

where

$$\bar{A}_v := \begin{bmatrix} -\frac{R}{L} & -\frac{1}{2L} & 0 & \frac{v}{2L} - K_1K_2 & -K_1 + \frac{R}{L} \\ \frac{1}{2C_0} & 0 & -\frac{v}{2C_0} & 0 & \frac{v-1}{2C_0} \\ 0 & \frac{v}{2L} & -\frac{R}{L} & -\frac{1}{2L} - K_1K_2 & -K_1 + \frac{R}{L} \\ -\frac{v}{2C_0} & 0 & \frac{1}{2C_0} & -\frac{2}{R_0C_0} & \frac{v-1}{2C_0} \\ 0 & 0 & 0 & -K_1K_2 & -K_1 \end{bmatrix}$$

$$
\bar{B}_v := \begin{bmatrix} \frac{2V_{in}}{L} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} A_v x_e(\lambda_e) \\ \hline 0 \end{bmatrix}
$$

$$
\bar{C} := [0\ 0\ 0\ 1\ 0].
$$

From this dynamic, we can formulate the robust hybrid system as follows:

$$
\bar{\mathcal{H}} : \begin{cases} \begin{bmatrix} \dot{\tilde{\chi}} \\ \dot{v} \\ \dot{\tau} \\ \dot{\chi}_{ae} \\ \dot{\lambda}_e \\ \dot{\chi}_e \end{bmatrix} = \begin{bmatrix} \bar{A}_v \tilde{\chi} + \bar{B}_v \\ 0 \\ 1 - dz(\frac{\tau}{T}) \\ \bar{A}_{\lambda_e} \chi_{ae} + \bar{\mathcal{B}} \\ f_1(V_r) \\ f_2(\lambda_e) \\ 0 \end{bmatrix}, & \bar{\zeta} \in \bar{\mathcal{C}} \\ \begin{bmatrix} \tilde{\chi}^+ \\ v^+ \\ \tau^+ \\ \chi_{ae}^+ \\ \lambda_e^+ \\ \chi_e^+ \end{bmatrix} \in \begin{bmatrix} \tilde{\chi} \\ \underset{i \in \{-1,1\}}{\operatorname{argmin}}\ \tilde{\chi}^\top \bar{P}(\bar{A}_i \tilde{\chi} + \bar{B}_v) \\ 0 \\ \chi_{ae} \\ \lambda_e \\ \chi_e \end{bmatrix}, & \bar{\zeta} \in \bar{\mathcal{D}}, \\ \tilde{y} = \bar{C}\chi \end{cases} \tag{27}
$$

where $\bar{\zeta} = [\tilde{\chi}\ v\ \tau\ \chi_{ae}\ \lambda_e\ \chi_e]^\top \in \bar{\mathbb{H}}$, such that, $\bar{\mathbb{H}} := \{\tilde{\chi} \in \mathbb{R}^5, v \in \{-1,1\}, \tau \in \mathbb{R}^+, \chi_{ae} \in \mathbb{R}^5, \lambda_e \in [\nu, 1-\nu], \chi_e \in \mathbb{R}^5\}$, with a parameter $\nu > 0$ small enough. Moreover, $\bar{A}_{\lambda_e} = \bar{A}_v$, where v is replaced by λ_e and $\bar{\mathcal{B}} := [\begin{smallmatrix} \mathcal{B} \\ 0 \end{smallmatrix}]$. Likewise, the flow and jump sets are

$$
\bar{\mathcal{C}} := \{\bar{\zeta} : \tilde{\chi}^\top \bar{P}(\bar{A}_v \tilde{\chi} + \bar{B}_v) \leq -\tilde{\chi}^\top \bar{Q} \tilde{\chi},\ \tau \in [0,T]\} \tag{28}
$$

$$
\bar{\mathcal{D}} := \{\bar{\zeta} : \tilde{\chi}^\top \bar{P}(\bar{A}_v \tilde{\chi} + \bar{B}_v) \geq -\tilde{\chi}^\top \bar{Q} \tilde{\chi},\ \tau = T\}. \tag{29}
$$

with matrices $\bar{P}, \bar{Q} \succ 0 \in \mathbb{R}^5$.

Theorem 3. *For a given output reference* $V_r = V_{\max}\sin(\omega t)$*, consider* $\chi_e(x_e(\lambda_e(V_r)))$*, being* $x_e(\lambda_e)$ *defined in Proposition 1 and some given parameters* T_e *and* T*, such that condition* (15) *is satisfied. Moreover, consider there exist matrices* $\bar{P} \in \mathbb{R}^5 \succ 0$*,* $\bar{Q} \in \mathbb{R}^5 \succ 0$ *and parameters* $\gamma^2 = \rho > 0$ *and* $\epsilon > 0$*, such that, all Lemma 4 assumptions are satisfied. Then, for any parameters* $X_t > 0$*,* $X > 0$ *and for*

$$\varepsilon_0(t) := \frac{\gamma^2}{2}\int_t^{t+T_e}\left\|\frac{d\chi_{ae}(\bar{t})}{d\bar{t}}\right\|^2 d\bar{t}.$$

and any scalar $T \in [0, T^*]$ *the following holds:*

1. *set*

$$\mathcal{A}_t := \{\zeta \in \mathbb{H} : \|\chi - \chi_e\| \le X + X_t, \tau \in [0, T]\}$$

   is compact,

2. *set* $\mathcal{A}_t$ *is UGAS for hybrid system* (27)–(29),

3. *the system output is bounded by* $\int_t^{t+T_e}|y(t) - V_r(\bar{t})|^2 d\bar{t} < T_e X^2 + \varepsilon_0$, (12)–(14),

4. *set* $\mathcal{A}_{t,0} := \{\zeta : \|\chi - \chi_e\| \le X_t, u \in \{0, 1\}, \tau \in [0, T]\}$ *is globally practically asymptotically stable for system* (12)–(14)*. As long as,* T *is small enough, set* $\mathcal{A}$ *can be arbitrarily close to* $\mathcal{A}_0$.

Proof. The proof is direct from Theorem 2 proof. □

Remark 4. Remark 3 makes reference to modelled errors, when considering $R_0 \ll R$. This can be compensated from the hybrid scheme (27)–(29) and applying Theorem 3. ⌟

Now, all items of Problem 1 are satisfied.

6. Simulations

In this section, we validate our hybrid control approach to the boost inverter (2) in simulations. These simulations are performed in MATLAB/Simulink by exploiting the HyEQ Toolbox [21].

The boost inverter parameters are given in Table 2, satisfying Assumptions 1. Moreover, we take $K_1 = 1$ and $K_2 = 1000$. Then, the related optimization problem of Theorem 3 provides $\epsilon = 9 \cdot 10^{-8}$, $\rho = 1.14 \cdot 10^{-5}$, and

$$\bar{P} = \begin{bmatrix} 3.25 & 0.00 & -0.11 & 0.00 & -3.11 \\ 0.00 & 0.15 & -0.00 & -0.05 & 0.00 \\ -0.11 & -0.00 & 3.25 & 0.00 & -3.11 \\ 0.00 & -0.05 & 0.00 & 0.15 & 0.00 \\ -3.11 & 0.00 & -3.11 & 0.00 & 6.23 \end{bmatrix}.$$

The upper bound of T for a chattering of $10A$ in the currents, $4V$ in the voltages and $2 \cdot 10^5$ in ρ is $T^* = 7.92 \cdot 10^{-4}$. Hence, the selected minimum dwell-time was $T = 10^{-6}s$.

Table 2: Simulation parameters

Parameter	Notation	Value	Units
DC input voltage	V_{in}	70	V
Reference peak voltage	V_{max}	$220\sqrt{2}$	V
Nominal angular frequency	ω	120π	rad/s
Nominal load resistance	R_0	100	Ω
Estimated series resistance	R	0.02	Ω
Inductor	L	44	mH
Output capacitor	C	200	λF

Note that Property 7 and condition (15) are satisfied. Some simulations are performed considering two scenarios:

6.1. Scenario I

This scenario is characterized by a perturbation of the load resistance, R_0, of 100% with respect to its nominal one, i.e., it changes from 100Ω to 200Ω at 0.02s. Figure 4 shows the evolution of the voltages, currents and variable ϱ of (2) when the controlled hybrid system (27)–(29) is applied. Note, after the change of load R_0 at 0.02s, the currents i_1 and i_2 converge to a different equilibrium. Moreover, Fig. 4 performs the system output. Note, the extra state ϱ evolves, such that, the system output gets to reject the perturbations and the modelled errors.

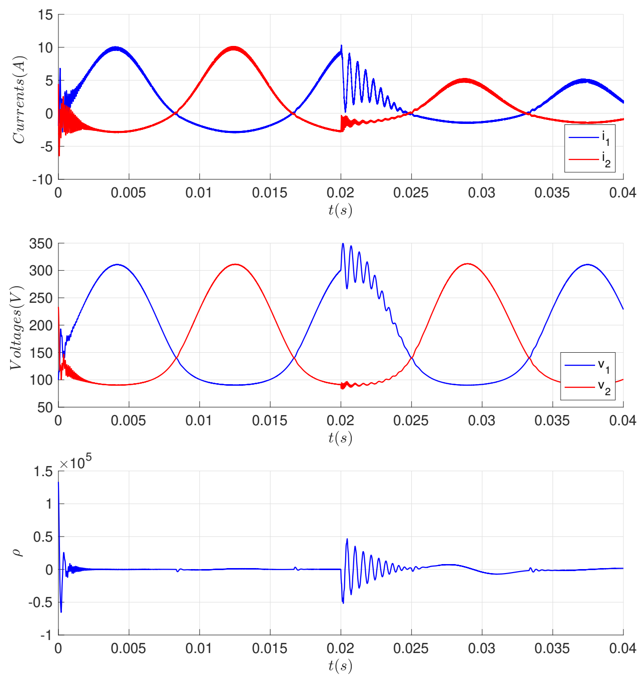

Figure 3: Evolution of the states with a 100% variation of R_0 w.r.t. its nominal one.

6.2. Scenario II

In this second scenario, the input voltage V_{in} changes a 30% with respect to its nominal value, i.e., from 48V to 100V at 0.02s. Figures 5 and 6 show the state and output system evolutions. Note, the system output rejects the perturbations and modelled errors with the introduced extra state ρ, converging $y(t)$ to $V_r(t) = V_{\max}\sin(\omega t)$ after the change of V_{in}. Nevertheless, we can

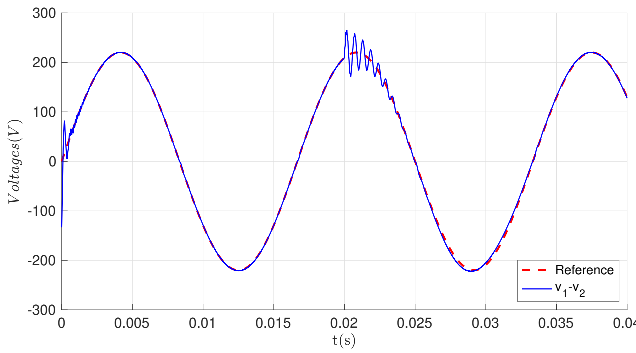

Figure 4: Output with a 100% variation of R_0 w.r.t. its nominal one.

conclude the error tracking system output is regulated, such that, is robust with respect to the considered perturbation.

7. Conclusion

In this paper, a hybrid control law is proposed for a DC-AC converter also known as a boost inverter. This control takes into account the real nature of the inverter signals, meaning the continuous-time dynamics represented by the voltages and the currents and the discrete-time dynamic, which is the discrete input signal. Moreover, a minimum dwell time is also considered. UGAS is guaranteed to an attractor, ensuring the convergence of $y(t)$ to a neighbourhood of its reference, small enough. An extension of this result is given to provide robustness properties in terms of error tracking output regulation. Here, it is shown the ability of the system output to reject any parameter change.

The future work is to obtain experimental results in a prototype.

Acknowledgements

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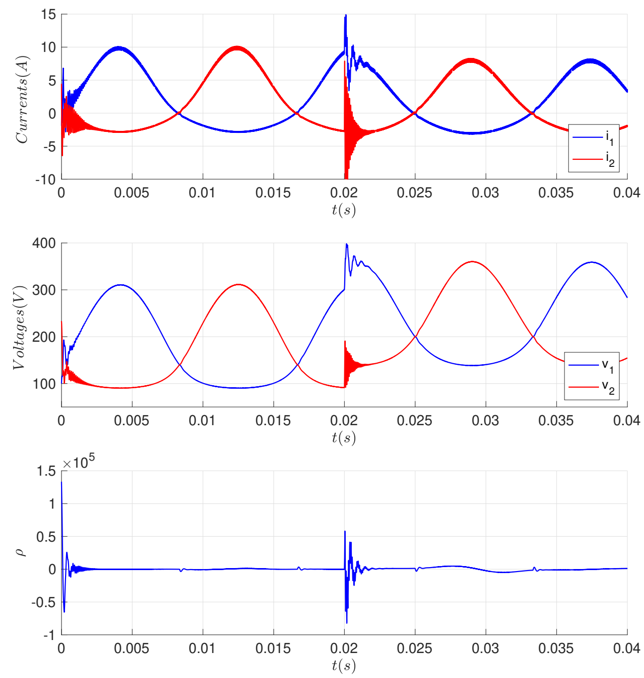

Figure 5: Evolution of the states with a 30% variation of V_{in} from its nominal one.

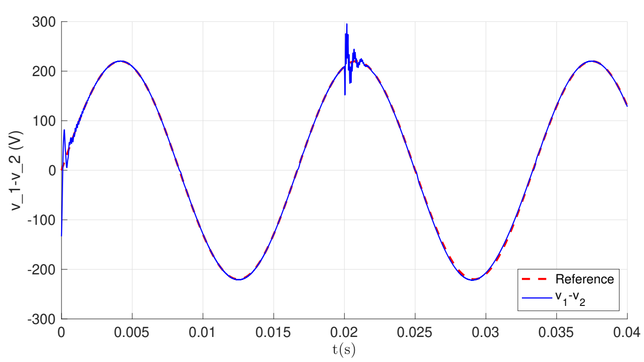

Figure 6: Output system with a 30% variation of V_{in} from its nominal one.

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