Non-Uniform Sampling Theory applied to FM Channel Optical Feedback Interferometry for Displacement Sensors

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***Abstract*—In this paper, the optical feedback interferometry (OFI) signals obtained from the frequency modulated (FM) channel are processed by a non-uniform sampling (NUS) method to retrieve target displacements. The FM OFI channel signal is converted into its corresponding amplitude modulated (AM) signal using a free-space Michelson interferometer as an edge filter with a sensitivity of 1.3 $(\mathrm{GHz})^{-1}$. The laser diode (LD) is operated in moderate optical feedback regime and its wavelength is modulated with a sinusoidal dithering signal applied on the LD driving-current. This dithering signal allows the recovery of sub half-wavelength displacements. Contrary to phase unwrapping methods, the proposed method based on NUS relies only on OFI's fringe detection to recover the target's displacement and thus is not affected by the non-linear response of the employed edge filter. In addition, compared to lock-in techniques, the dynamic range is not limited by the tuning range of the LD. Using a laser diode emitting at 1550 nm, the noise equivalent displacement for a 1 kHz bandwidth is approximately 8.4 nm and 0.9 nm for the AM- and the FM-channel, respectively.**

***Keywords*—optical feedback interferometry; self-mixing; OFI FM-to-AM conversion; non uniform sampling; Mach Zehnder**

I. Introduction

Optical feedback interferometry (OFI) has been widely investigated as it results in a self-aligned and cost-effective sensing system for displacement sensors for instance [1]–[3]. In OFI, a portion of the laser beam can be back-scattered from a target placed away from the laser and can thus re-enter the active laser diode (LD) cavity. This optical feedback directly interferes with the optical field present inside the LD cavity, resulting in the modulation of both amplitude (AM) and frequency (FM) of the optical field, modelled by Lang and Kobayashi equations [4]. The OFI information embedded in the FM OFI channel was shown to benefit from a signal-to-noise ratio approximately two orders of magnitude better than the AM OFI one [5]. As a result, FM OFI signals provide an excellent opportunity to improve OFI sensitivity for both low-noise small displacement applications and low back-scattered OFI signals due to target surface diffusive property.

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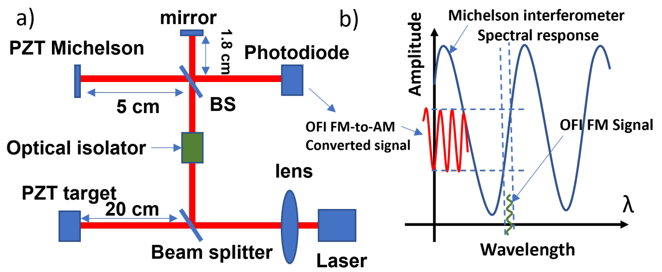

Fig. 1. Schematic diagram of: (a) the OFI system with OFI FM-to-AM conversion using a Michelson interferometer with a piezo-electric transducer (PZT) as target, and (b) OFI FM-to-AM conversion via the steep edge of the Michelson interferometer response used as a filter.

To exploit the FM OFI channel and to easily access the embedded FM information, direct conversion of the FM OFI modulation to light intensity amplitude modulation (AM) is desirable. Optical filters can be used for this purpose, as shown in Fig. 1 b). The first practical conversion has been achieved using the edge of the acetylene gas cell optical filter [6]. The steep optical absorption profile of the cell provides an OFI FM-to-AM conversion factor of $\approx 2.2\,(\mathrm{GHz})^{-1}$. To improve further both the sensitivity and the flexibility of the filter, approaches based on Mach Zehnder interferometers (MZI) have then been proposed [5], [7]. In addition, contrary to the gas cell filter, the MZI OFI FM-to-AM conversion sensitivity can be adjusted by the optical path length imbalance between the two MZI arms. As a result, a 19 $(\mathrm{GHz})^{-1}$ sensitivity was achieved in [5], with a noise equivalent displacement (NED) spectrum density of 1.3 pm/$\sqrt{Hz}$.

In this work, we demonstrate that dithering and non-uniform sampling (NUS) methods can successfully retrieve target displacements from the OFI FM-to-AM signal. Contrary to lock-in techniques [5] and phase unwrapping methods [8], it can both extend the dynamic range further and be less sensitive to any OFI FM-to-AM conversion non-linearities. In addition, to allow wavelength dithering and to improve further the system resilience regarding temperature- and parasitic mechanicaldrift, the implemented path-length imbalance of a free-space Michelson interferometer (MI) used as edge filter is set to be 6.4 cm corresponding to a $1.3\,(\mathrm{GHz})^{-1}$ sensitivity (Fig. 1 a)).

This paper is organized in two main parts. First, in Section II, an overview of the FM OFI signal, the proposed OFI FM-to-AM conversion and displacement recovery method are presented. In Section III, the experimental setup and results are shown followed by a conclusion in section IV.

II. FM OFI CHANNEL

A. FM OFI overview

For the AM channel, the OFI modulates the optical output power (OOP) of the LD, denoted $P(t)$, given by [1]:

$$P(t) = P_0\left[1 + m\cos\left(\Phi_F(t)\right)\right], \tag{1}$$

where P_0 is the emitted OOP under free-running conditions, m is the modulation index and $\Phi_F(t)$ is the laser output phase in the presence of feedback. $\Phi_F(t)$ can be directly expressed as a function of the laser frequency ν_F as follows:

$$\Phi_F(t) = 2\pi\nu_F(t)\,\tau_{ext}(t), \tag{2}$$

where $\tau_{ext} = 2D(t)/c$ is the laser-target external round trip time of flight with $D(t)$ the laser-target distance and c the speed of light. In presence of optical feedback, ν_F is related to the unperturbed LD frequency ν_0 by [1], [2]:

$$\left[\nu_0(t) - \nu_F(t)\right]\tau_{ext} = \frac{C}{2\pi}\sin\left(2\pi\nu_F(t)\,\tau_{ext} + \arctan\alpha\right) \tag{3}$$

where α is the linewidth enhancement factor of the LD and C the optical feedback factor. Depending on C, the laser can operate in different regimes. OFI sensing is generally performed under weak- ($C < 1$), moderate- ($1 < C < 4.6$), or strong- ($C > 4.6$) feedback regime. In the moderate feedback regime, the saw-tooth shaped OFI fringes [9] intrinsically provide motion direction indication and fringes are more easily detected. As a result, this regime is usually preferred as it enables more efficient processing [10].

In open-loop configuration, the design of the optical filter should take into the maximum span $|\Delta\nu|$ to avoid folding of the FM converted OFI signals. This folding can result in inverting fringe directions and thus corrupt the reconstructed displacement if not detected. From (3), the variation of the LD frequency $\Delta\nu$ is strongly constrained as:

$$|\Delta\nu| \leq \frac{C}{2\pi\tau_{ext}}. \tag{4}$$

Note that in the closed-loop approach based on fringe-locking to the half fringe, this range could be further extended and could therefore allow the design of even steeper filters if temperature and mechanical induced drifts on the filter are efficiently removed [5].

B. The OFI FM-to-AM conversion

As previously mentioned, the OFI FM-to-AM conversion is performed using a MI shown in Fig. 1. Compared to a MZI, it is more compact as the imbalance arm is divided by two and requires less optical components which eases alignment. The output power can be expressed as:

$$P_{out} = P_{in}\left[1 + \cos\left(2\pi n(\nu)\frac{\Delta d_{path}}{c}\nu\right)\right], \tag{5}$$

where P_{in} is the injected light power in the MI, Δd_{path} the MI path difference length, n the refractive index of the optical path. As mentioned in [5], the maximum sensitivity S is achieved at the middle of the transmission power response of the MI. It is given as:

$$S = \frac{1}{P_{in}}\frac{\partial P}{\partial\nu} = 2\pi n_g(\lambda)\frac{\Delta d_{path}}{c}, \tag{6}$$

where $n_g(\lambda) = n(\lambda) - \lambda\frac{\partial n(\lambda)}{\partial\lambda}$ is the group index. In the case of open-loop OFI approach, it is necessary to provide a mean to estimate the achievable OFI FM-to-AM conversion range. Similarly to bandwidth definition, it can be defined as the range for which the sensitivity is reduced by a factor of $\sqrt{2}$. It can be shown from (6) that the available conversion range $\delta\lambda$ can then be approximated by:

$$\delta\lambda \approx \frac{\lambda^2}{4n\Delta d_{path}}. \tag{7}$$

This conversion range should cover not only the LD wavelength variation induced by OFI but also wavelength variation induced by current modulation required for dithering [8]. There is thus a trade-off between sensitivity and conversion range for open-loop configurations.

C. Non-Uniform Sampling

In moderate feedback regime, it was shown in [8] that OFI can be interpreted as an inherent non-uniform sampling system with its own embedded phase level-crossing detector. By monitoring the OOP discontinuities, a phase domain level crossing every 2π can thus be obtained from which the target displacement can be retrieved. In a manner similar to approaches used in NUS analog-to-digital converters [11], a phase dither can be added to the phase Φ_0 via the LD wavelength modulation, so that both the number of crossed levels as well as the rate of level crossings can be increased. Thus, in our case, dithering causes occurrence of virtual-fringes in the OFI signal. This is particularly relevant for recovering small amplitude displacements.

III. EXPERIMENTAL RESULTS

A. Experimental set-up

The schematic diagram shown in Fig. 1 describes the experimental set-up where as Fig. 2 is the actual set-up. As light source, a DFB LD emitting at $\lambda = 1550$ nm is used with a total output power set at 11 mW for a dc current bias of 24 mA. In addition, the LD is modulated at a dithering frequency $f_d = 1$ kHz with a current amplitude of approximately 1.25 mA to generate the dithering. A 50/50 beam splitter (BS) and an optical isolator are used to separate (1) the sensing branch which connects the laser to the target, from (2) the OFI FM-to-AM conversion based on the MI with an imbalance path $\Delta d_{path} = 6.4$ cm, which corresponds to a sensitivity of

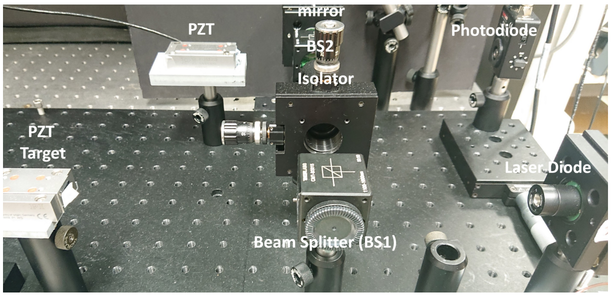

Fig. 2. Experimental setup of the OFI system with OFI FM-to-AM conversion using a Michelson interferometer.

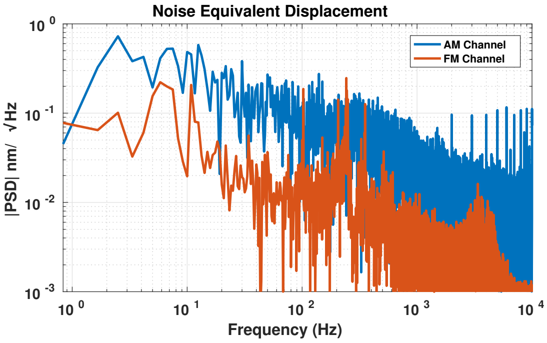

Fig. 3. NED power spectrum density for the AM- and FM-channel of the designed OFI system.

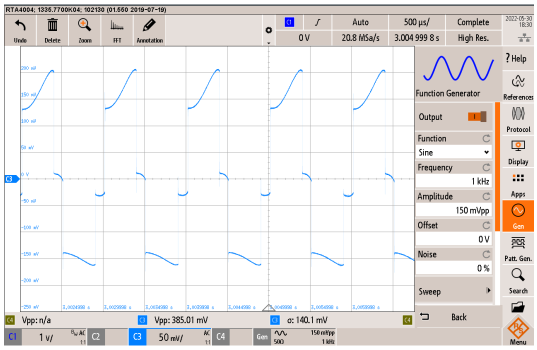

Fig. 4. OFI FM channel signal after OFI FM-to-AM conversion without any target motion and with a wavelength dithering signal at 1 kHz.

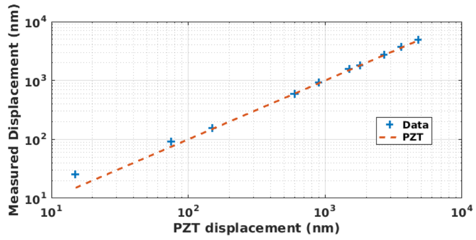

Fig. 5. NUS approach based displacement reconstruction of an experimental target vibrating at 30 Hz while using a wavelength dithering signal at 1 kHz.

1.3 (GHz)$^{-1}$ and a 37 pm free spectral range. Two piezoelectric transducers (PZT) from Physic Instrumente are used: (1) as the target and (2) to tune the MI edge filter. The OFI FM-to-AM converted output of the MI is monitored by a photodiode module PDA10CS2 from Thorlabs. For comparison, the LD monitoring photodiode is also directly recorded. The data is acquired by a Rohde & Schwarz RTA4004 oscilloscope at 10 MSample/s.

B. Noise performances

The NED can be estimated as:

$$NED = \frac{\lambda}{2}\frac{V_{RMS}}{V_{pp}}, \tag{8}$$

where V_{RMS} is the OFI signal RMS noise value and V_{pp} the OFI signal amplitude. The plots shown in Fig. 3 compare the NED power spectrum density achieved with the FM- and the AM-channel when the target remains still. For the AM and FM channels, the NED$_{AM}$ and NED$_{FM}$ are approximately 8.4 nm and 0.9 nm for a 1 kHz bandwidth respectively. In Fig. 3, a burst of noise around 240 Hz is observed for the FM-channel. It is related to a mechanical resonance of the PZT-Michelson used to set the MI edge filter at half-fringe to achieve the highest FM-to-AM conversion sensitivity (see Fig. 1).

C. Displacement reconstruction

The applied dithering signal induces 2 fringes (see Fig. 4). Note that contrary to the AM channel, the demodulated FM channel is less affected by the LD output power modulation than the AM channel. The modulation represents only $\approx 8\%$ and $\approx 1700\%$ of the output signal for the demodulated FM- and AM-channel, respectively. As a result, a simple fringe detection algorithm (using the derivative of OFI signal) is applied on the recovered OFI signal from the FM channel to generate the non-uniform sample set that is then processed by the NUS method using the spline as interpolator to reconstruct the displacement. Fig. 5 shows the measured displacement amplitude extracted from the OFI signals acquired during 1.2 s. The sub-$\lambda/2$ displacement of the target vibrating at 30 Hz can be retrieved using the NUS with the 1 kHz dithering signal.

IV. Conclusion

We have demonstrated that NUS can also be applied to OFI FM-to-AM conversion for OFI applications. Similar to the AM channel, it is shown that using a dithering signal allows to retrieve sub-$\lambda/2$ displacements using the FM channel while generating only 2 virtual-displacement fringes. The achieved NED with the FM- and the AM-channel is approximately 0.9 nm (with a 1.3 (GHz)$^{-1}$ conversion gain) and 8.4 nm, respectively. Note that the presented work is a proof-of-concept of use of non-uniform-sampling for measurement retrieval from the FM OFI channel, and future work will concentrate on implementing this optical processing in an integrated photonic chip.

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